COALITION MODELING: FROM SPIN GLASSES TO HUMAN SOCIETIES

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Coalition modeling: from spin glasses to human societies

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Abstract

The first part of this dissertation aims to provide a synopsis of the currently existing theoretical work on the use of spin glasses to study the formation of coalitions among actors. In this framework, a system of actors is mapped onto a network of interacting spins. The spin value of an actor is chosen in such a way that it attains the most "comfortable" position, i.e. the one that minimizes the actor's individual energy. The spin value also determines which coalition this actor is part of. In contrast to physics, where spins can only evaluate the immediate energy benefit of a change in spin value, an actor has a long term vision. This is called extended rationality. Thanks to this extended rationality, the actor can see a minimization of its individual energy through sub-optimal intermediate system configurations. The dynamics of two specific models of coalition formation will be studied in detail: The Natural model of coalition forming and the Global alliance model, both established by Vinogradova and Galam in a series of papers [8, 9, 10]. These are, in turn, built upon the Edwards and Andersons and the Mattis spin glass models. Some additional extensions to these models, introducing diplomatic interactions between two actors and risky behaviour of an actor are also discussed. These extensions are built upon the physical concepts of magnetic fields and temperature.

In the second part of this work, the aim is to develop software for simulating Natural model systems in order to study their stability and model real life cases. To test this software, we simulate the systems put forward in the first part as examples. It is shown that the dynamics of the Natural model and the Global alliance model are indeed present in the simulation software. The stability of the Global alliance model under various circumstances is studied as well. We show that the Global alliance model becomes increasingly unstable when more global principles and actors are introduced. Finally, we propose a way the framework of the Natural model and its computational implementation could be used to make predictions with regard to the outcome of upcoming presidential elections in the United States.

Extended Abstract

Het doel van deze masterproef is een overzicht bieden van de huidige theorieën over de vorming van coalities tussen verschillende actoren gebaseerd op spinglasmodellen. Binnen het raamwerk van een spinglasmodel wordt een systeem van actoren geprojecteerd op een netwerk van interagerende spins. De spinwaarde die een actor bezit is gekozen vanuit het perspectief van de meest 'comfortabele' positie. Hierbij bedoelt men een positie waarbij de individuele energie van deze actor minimaal is. Deze positie bepaalt tevens tot welke coalitie de actor behoort. In tegenstelling tot de fysische systemen, waar de spins enkel hun onmiddellijk energetisch voordeel, als gevolg van een veranderende spinwaarde kunnen berekenen, hebben actoren een langetermijnvisie. Deze wordt ook wel 'lange rationaliteit' genoemd. In dit geval kan een actor zijn laagste individuele energie bereiken ondanks sub-optimale tussenliggende systeemconfiguraties. Twee specifieke modellen rond coalitievorming worden in detail besproken, namelijk van 'The Natural model of coalition forming' en 'The Global alliance model'. Beiden zijn geïntoduceerd door Vinogradova en Galam in een reeks van essays [8,9,10]. Deze modellen zijn op hun beurt gebaseerd op zowel het Edwards en Andersons model als op het spinglas model van Mattis. Ook komen een aantal uitbreidingen op deze modellen, die ontstaan door de projectie van menselijke interacties, aan bod. Zo zijn er bijvoorbeeld diplomatische interacties tussen verschillende actoren en het riskante, gedrag van een bepaalde actor. Voor deze uitbreidingen wordt er gekeken naar de fysische concepten van magnetische velden en temperatuur.

Het tweede deel van deze masterproef behandelt de ontwikkeling van een simulatieprogramma dat de stabiliteit van systemen bestaande uit actoren bestudeert en waarmee reële voorbeelden gemodelleerd kunnen worden. We testen deze software aan de hand van de voorbeeld systemen uit het eerste deel. De resultaten van deze tests tonen dat de eigenschappen van het Natural model of coalition forming inderdaad aanwezig zijn in de simulaties. Daarnaast wordt de stabiliteit van Global alliance model systemen onder verschillende omstandigheden bestudeerd. We tonen aan dat Global alliance model systemen instabieler worden wanneer meerdere globale principes van toepassing zijn. Als afsluiter werpen we een blik richting de toekomst. We tonen hoe het Natural model en de ontwikkelde software dienst kunnen doen als basis voor het voorspellen van de resultaten van de volgende presidentiële verkiezingen in de Verenigde Staten van Amerika.

Content

Part I Theory

Introduction to coalition modeling

A lot of words have already been written about the theory of spin glasses and their various applications. This work borrows from these the key concepts of spin glass theory needed for the purpose of modeling social interactions. Modeling social interactions through spin glasses is certainly not new. It was tried first by Axelrod and Bennet in [5]. This attempt serves as a cautionary tale as the model contained many flaws as was pointed out by Galam in [6]. One cannot simply map social terms and dynamics onto physical concepts and models. A few years later, Vinogradova and Galam established the Natural model of coalition forming and the Global alliance model in a series of papers [8, 9, 10]. We will study these models and the dynamics they contain, as well as some extensions, before we try to implement them into a simulation tool. During this work, various illustrations will be given. These are inspired by the examples in the works of Vinogradova and Galam as well as the textbook from the course 'History of world politics' at the University of Ghent. This work came about during the COVID-19 quarantine and I have found that, while playing with friends, the game of Diplomacy by Allan B. Calhamer is, accidentally, a very good implementation of the Natural model in board game format and can serve as an excellent way to grasp the intricacies of the model. Finally, we recognize that the potential of the Natural model and its extensions is almost limitless. The Natural model is shown to model the stability of the Eurozone in [9] and helps explain the stability in Europe after the collapse of the Soviet Union [10]. We hope to add a new use-case to this list by proposing a way the model could be used to make predictions about the presidential elections of the United States.

PART I THEORY

Chapter 1 Spin glass physics

1.1 Introduction to spin glasses

In this chapter, we introduce spin glasses and the physical concept of frustration. In later chapters, we will use spin glass models and concepts introduced here to model social interactions between various actors. Throughout this work we will talk about lattices and lattice sites as they are the framework upon which spin glasses are built. An example of a lattice is shown in Figure 1.1. A lattice can also be interpreted as a graph G where every vertex i and its four neighbouring vertices j are connected by an edge representing a coupling constant, J_{ij}. We start with the trivial model of a rectangular lattice with the coupling constants assumed to be positive and equal for every lattice site pair i, j. From there on, we loosen the restrictions on the coupling constants, creating ever more disorder in the model. It is also important to note that for the remainder of this work, we will assume our physical models to be at a temperature of $T = 0$. This allows for major simplifications in the dynamics of spin glass systems. Finally, two spin glass models that will prove useful in later chapters are discussed as well.

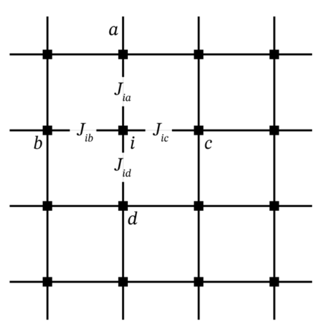

Figure 1.1: An example of a lattice. Lattice sites i (full squares) are connected to their neighbours a, b, c and d with coupling constants J_{ia}, J_{ib}, J_{ic} and J_{id} (full lines) respectively.

1.2 Spin glass mechanics

One can think of a spin glass system as a lattice on which each of the N lattice sites possesses a variable called spin. The spin σ_i of lattice site i can take the values $\sigma_i = +1$ and $\sigma_i = -1$ and is dependent on the spin values of the neighbouring lattice sites j through a Hamiltonian such as

$$H(\{\sigma_i\}, \{J_{ij}\}) = -\sum_{<i,j>}^{N} J_{ij}\sigma_i\sigma_j, \tag{1.1}$$

where J_{ij} is the coupling constant between lattice sites i and j and the summation extends over all nearest-neighbours lattice site pairs. We can see that for the case where $J_{ij} = J$ for all i, j with J strictly positive, this Hamiltonian is identical to the Hamiltonian of a ferromagnetic Ising system in absence of a magnetic field [2]. An example of such a system is shown in Figure 1.2A while a possible system configuration is shown in Figure 1.2B. Using the Hamiltonian from Equation (1.1), we can work out that this system configuration has a system energy $E = 0$. There are 2^N system configurations in total and for the sake of later discussions, these are listed in Figure 1.3. In Figure 1.3, one can also see that every system configuration has an inverse, i.e. a system configuration in which all spin values are inverted ($+1$ becomes -1 and vice versa). A system configuration and its inverse are topologically identical, meaning there are only 2^{N-1} topologically unique system configurations.

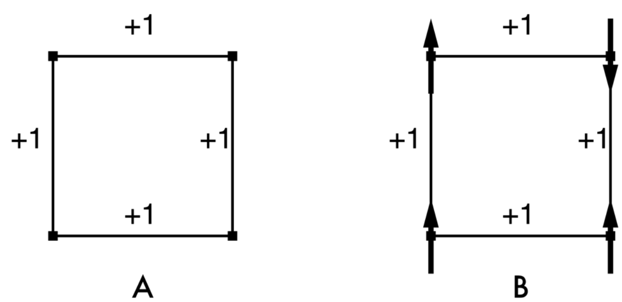

Figure 1.2: Panel A shows an example of a spin glass system with $J_{ij} = 1$ for all i and j. Panel B shows a system configuration of this system. The arrow on each lattice site represents the spin value on that lattice site. An up-arrow represents a spin value of $\sigma = +1$ while a down-arrow represents $\sigma = -1$.

Since they are topologically identical, a system configuration and its inverse share an energy level. This property allows us to simplify Figure 1.3. We do so by introducing the concepts of satisfied and unsatisfied couplings. A coupling between lattice sites i and j is called satisfied if the product $J_{ij}\sigma_i\sigma_j$ is positive. If it is negative, we call it unsatisfied. We can see that for the ground state configurations of the system shown in Figure 1.2, i.e. the configurations in Figure 1.3A with a system energy $E = -4$, all the couplings are satisfied while the couplings of the highest energy configurations (Figure 1.3B with a system energy $E = 4$) are all unsatisfied. From the other system configurations in Figure 1.3, we can see that a satisfied or unsatisfied coupling, just like the system energy, is invariant under inversion of spin values. This allows us to represent both a system configuration and its inverse with a single topological diagram in which the satisfied couplings are represented with solid lines and unsatisfied couplings with dashed lines. This leads, for the system shown in Figure 1.2 and its configurations in Figure 1.3, to the diagrams shown in Figure 1.4.

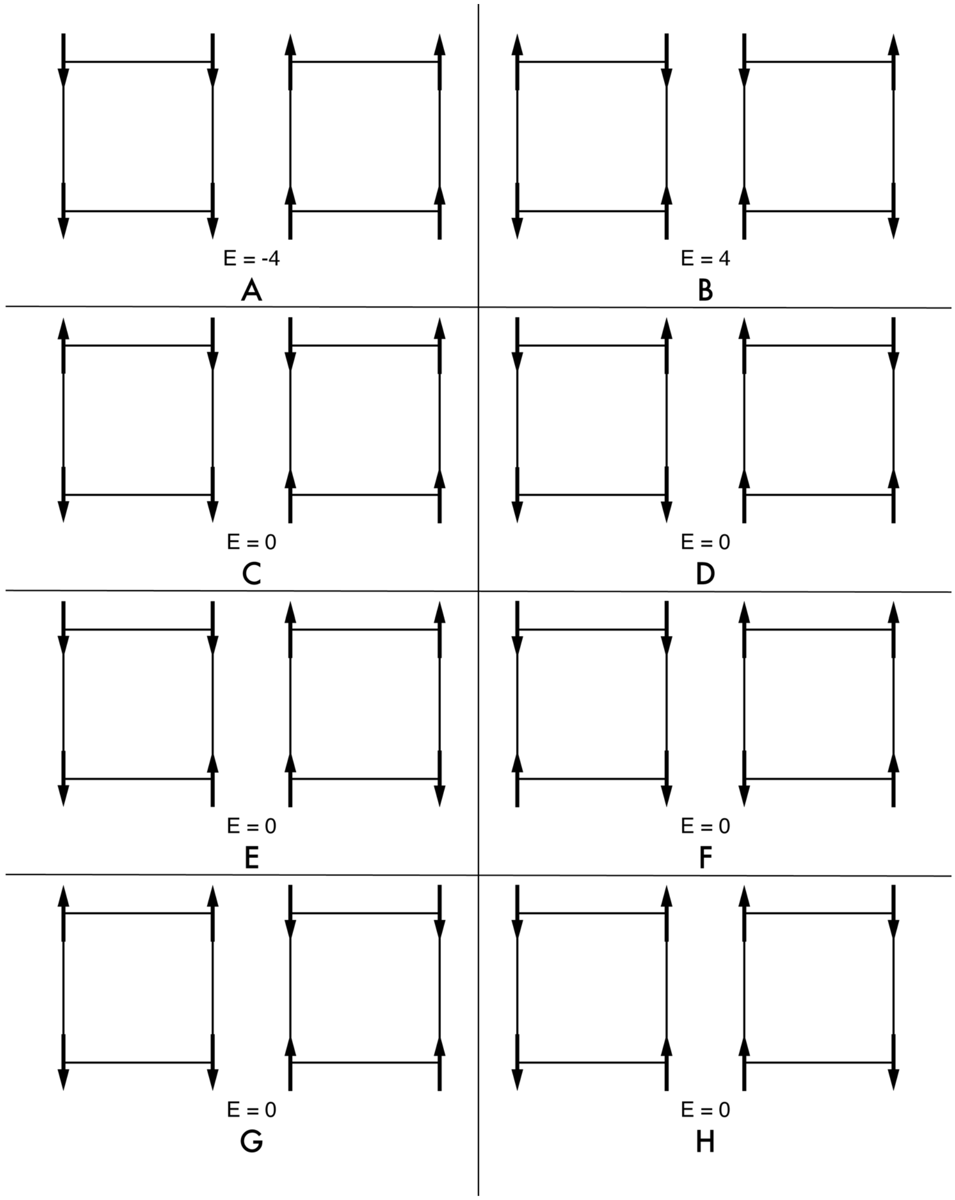

Figure 1.3: The sixteen possible permutations of the system shown in Figure 1.2. Configurations which are topologically identical (i.e. have the same system energy) are grouped together. Panel A shows the two configurations with the lowest system energy. These have all couplings satisfied and are called the ground states. Panel B shows the configurations with the highest system energy. All couplings are unsatisfied in this case. Panels C - H show configurations with two unsatisfied couplings

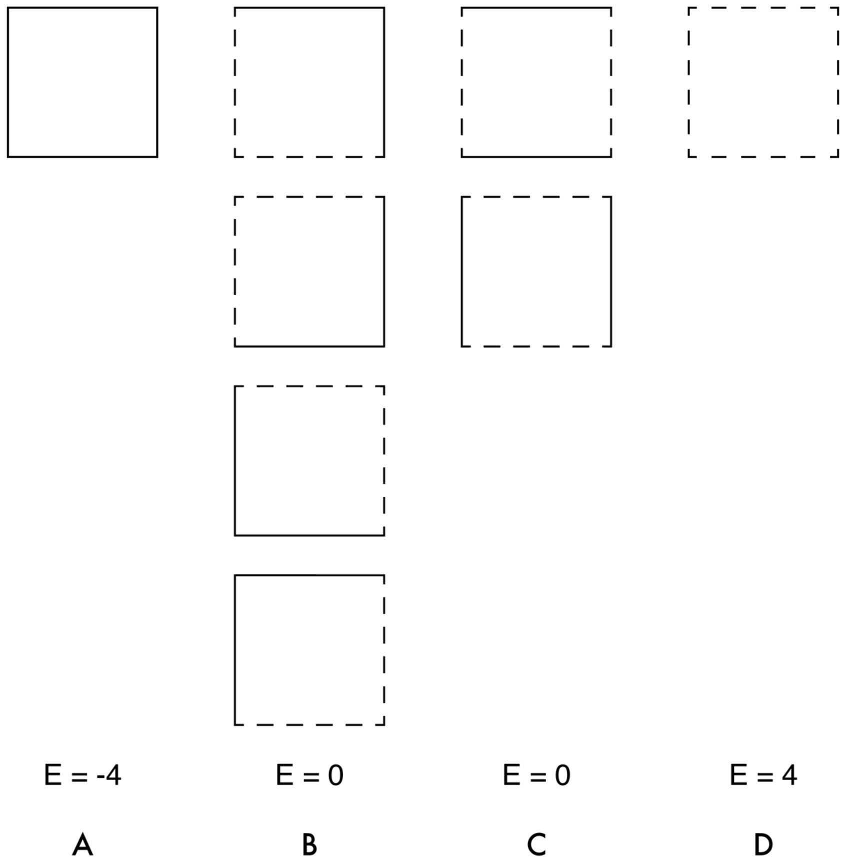

Figure 1.4: The sixteen system configurations of Figure 1.3 can be simplified into these eight, topological diagrams. Panel A represents the two ground states with all couplings satisfied. Panels B - C represent the twelve configurations with only two couplings satisfied. Panel D represents the two highest energy configurations where all couplings are unsatisfied. In a diagram, solid lines represent satisfied couplings while dashed lines represent unsatisfied couplings.

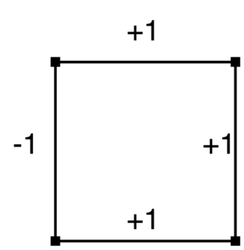

Figure 1.5: An example of a spin glass system where the sign of the couplings J_{ij} is randomly chosen.

1.3 Disorder and frustration

In a spin glass, the coupling constants between lattice sites are not strictly positive like in the example in Figure 1.2. They are randomly chosen to be positive or negative. A positive coupling will try to align the spins it connects (so both lattice sites have the same spin value): when both spin values are identical, the two lattice sites share a positive product $J_{ij}\sigma_i\sigma_j$ and the system energy will be lower than when the two lattice sites have different spin values. In the same manner, a negative coupling tries to anti-align the spins it connects (so both lattice sites have a differing spin value). The combination of these two dynamics results in a disordered lattice. For this reason, we call spin glasses disordered systems. Actually, we call them quenched disordered systems because the couplings between the neighbouring spins are constant on the time scale of spin fluctuations [1]. Figure 1.5 shows a system where the sign of the couplings is randomly chosen. There are, once again, $2^N = 16$ system configurations. These are the same as in Figure 1.3 but due to the negative coupling in the system, the topological diagrams are entirely different. One can immediately see that there is no diagram where all the couplings are satisfied. Instead, there are two quadruplets of topological equivalent diagrams. One of those quadruplets is the degenerate ground state quadruplet with energy $E = -2$ (Figure 1.7A). In other words, the ground state of this system is fourfold degenerate.

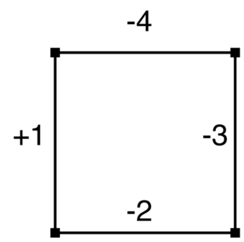

Figure 1.6: An example of a spin glass system with randomly chosen couplings J_{ij} (in sign and size).

The inability of a system to find a configuration that satisfies all the couplings between the spins is called frustration and such a system is called a frustrated system [2]. Because a diagram where all couplings are satisfied is topologically unique (i.e. there is only one such a diagram), this can also be phrased as: Frustrated systems are systems for which there is no single ground state diagram (i.e. the ground state is degenerate) [2]. A necessary condition for a frustrated system can be defined by interpreting a system configuration as a graph G in which the edge i, j between the vertices, or lattice sites, i and j represents the interaction coupling J_{ij}. Graph theory defines an n-mode cycle C_n as a non-empty path of length n along the lattice sites in which the only repeated vertices are the first and last vertices. A system is frustrated if there exists at least one cycle for which the following holds: If we fix an initial spin and try to chain-fix the other spins one after the other according to the signs of the couplings, we have to flip the initial spin when we return to it [2]. This condition for frustration is equivalent to the following mathematical condition: If there exists an n-mode cycle C for which the product of all the couplings J in the n-mode cycle is negative, then the system is frustrated [2].

$$\prod_{i,j \in C} J_{ij} < 0 \tag{1.2}$$

Take, for example, the system in Figure 1.5. The system in itself is also a 4-mode cycle. We can start from the bottom left and choose this spin to be $\sigma = +1$. Moving clockwise we can fix the spins such that the coupling between this spin and the previous spin is satisfied but upon returning to the bottom left we have to invert, or flip, this spin to satisfy its coupling with the bottom right spin. In another round of chain-fixing the spins, we have to flip all the spins and upon returning to the bottom left spin, we have to flip it once again.

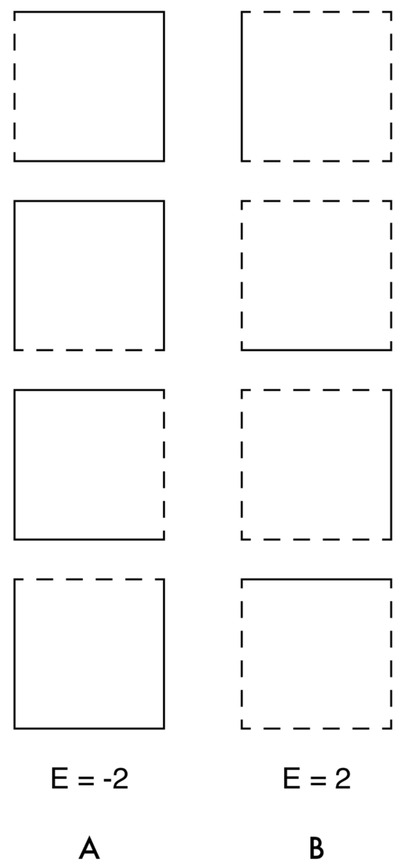

Figure 1.7: The topological diagrams of the system in Figure 1.5. They can be grouped into two quadruplets with respective energy E = -2 and E = 2. The four diagrams in a quadruplet are topologically equivalent: They have the same energy and can transition into one another by rotation of the system over 90 degrees. Quadruplet A is the one with the lowest energy: it is the ground state quadruplet.

This carousel continues and no configuration where all couplings are satisfied can be found. Since a coupling is shared between two spins, the unsatisfied coupling is able to propagate through the system [2]. For the system in our example, this means that it will spend the majority of its time fluctuating between the different ground state diagrams shown in Figure 1.7A. The necessary condition for frustration (1.2), however, is not sufficient because the coupling constants in spin glasses are most of the time not only random in sign but also in size. This can lift the degeneracy responsible for the frustration and as such, transition the frustrated system into a non-frustrated system. For an example of such a system, see Figure 1.6. The different topological diagrams are shown in Figure 1.8. The degeneracy between the topologically equivalent diagrams in the quadruplets is lifted and there is only one diagram with the lowest system energy $E = -8$. Even though there is no diagram where all couplings are satisfied and the product of all couplings is negative, the system is not frustrated. Note that the disorder in the system is inherently present in the Hamiltonian of the spin glass system through the set of coupling constants $\{J_{ij}\}$ that allow the lattice sites i to interact with neighbours j: $H = H(\{\sigma\}; \{J_{ij}\})$.

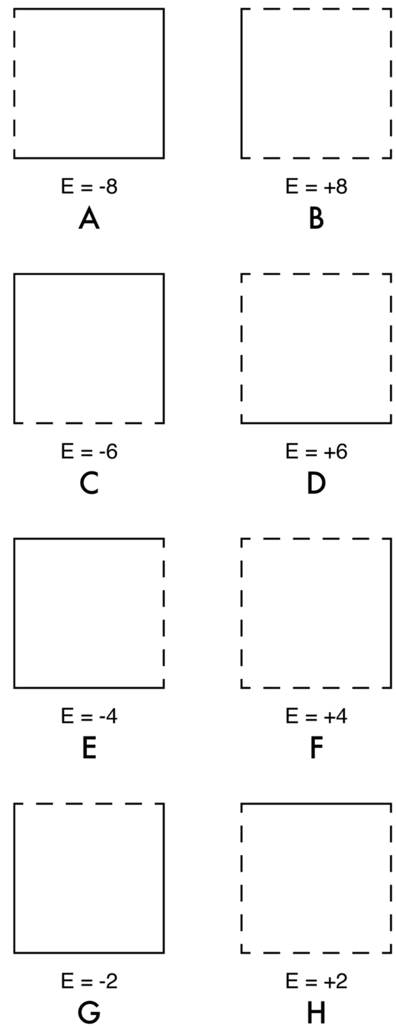

Figure 1.8: The topological diagrams of the system in Figure 1.6. The degeneracy between the diagrams is lifted due to the difference in size of the couplings. Diagram A represents the unique ground state diagram of the system.

1.4 Spin glass models

1.4.1 Edwards and Andersons model

The Edwards and Andersons spin glass model is a random-coupling spin glass model on a *d*-dimensional lattice with *N* lattice sites. Each lattice site *i* possesses a discrete variable, the spin σ_i, which can take the values $\sigma_i = +1$ or $\sigma_i = -1$. The coupling constant J_{ij} between the lattice sites *i* and *j* is randomly chosen from a Gaussian distribution $P(J_{ij})$. The energy of a system configuration $\Sigma = \{\sigma_i\}$ is given by the Hamiltonian

$$H(\{\sigma_i\};\{J_{ij}\}) = -\sum_{<ij>}^{N} J_{ij}\sigma_i\sigma_j, \tag{1.3}$$

where the summation extends over all pairs of nearest neighbour lattice sites[1]. This model can be solved exactly for the critical temperatures by means of the replica method [1]. A glassy phase with vanishing magnetization is discovered below this critical temperature [2]. This is however not in the scope of this work.

1.4.2 Mattis model

The Mattis model first introduced by Mattis (1976) is a random-site spin glass model [3]. Consider again a *d*-dimensional lattice. Each lattice site *i* has a spin $\sigma_i = \pm 1$ assigned to it and a variable ε_i. Just like the spin variable, ε_i can take the values $\varepsilon_i = +1$ or $\varepsilon_i = -1$. The coupling constant J_{ij} between lattice site *i* and *j* is written as

$$J_{ij} = J(R_{ij})\epsilon_i\epsilon_j, \tag{1.4}$$

where R_{ij} is some distance function between lattice sites *i* and *j*. The set $\{\varepsilon_i\}$ is randomly chosen from a probability distribution $P(\varepsilon_1, ..., \varepsilon_N)$ and is quenched compared to spin fluctuations. To understand this, we can compare each lattice site to a quark. Not only has a quark an electric charge with which it interacts with the electromagnetic force, it also has a colour charge with which it can interact with the strong force. In our case, the ε_i and ε_j of some lattice sites *i* and *j* interact with each other to form the coupling constant that acts between the lattice sites. The spins σ_i and σ_j of these lattice sites interact with each other and the coupling constant J_{ij} in the Hamiltonian of the system. In this case, Equation (1.1) can be recast as

$$H(\{\sigma_i\},\{\epsilon_i\};\{J(R_{ij})\}) = -\sum_{<ij>}^{N} J(R_{ij})\epsilon_i\sigma_i\epsilon_j\sigma_j, \tag{1.5}$$

where the summation extends over all pairs of nearest neighbours. By defining a new variable $\tau_i = \varepsilon_i\, \sigma_i$, the Hamiltonian becomes

$$H(\{\tau_i\};\{J(R_{ij})\}) = -\sum_{<ij>}^{N} J(R_{ij})\tau_i\tau_j. \tag{1.6}$$

If we choose the function $J(R_{ij})$ in such a way that it is always strictly greater than zero, this Hamiltonian resembles the Ising model Hamiltonian in absence of a magnetic field. Keeping in mind the frustration condition Equation (1.2), this spin glass model does not contain frustration [2].

1 We omit the external magnetic field-term for now. We will reintroduce it in Chapter 3.

Chapter 2 Natural model of coalition forming

2.1 Introduction to the Natural model of coalition forming

While social interactions are a complex topic, the principle of 'an enemy of my enemy is my friend' [4], allows a drastic reduction to a binary choice in the study of coalition forming. For instance, during the Second World War, a country had to choose between joining the allies or joining the axis. This binary choice can be thought of as choosing between two coalitions and a two coalition-system can be mapped onto a spin glass where each actor is a lattice site and the two possible coalitions are represented by the two values of the spin variable. What rests is to define a graph topology, that fits the problem. i.e. how many neighbours each actor has and with which coupling constant two neighbours are connected. The United States, for example, has diplomatic ties with several countries, meaning it has several neighbours on the lattice as well. The first attempt to implement this program was done by Axelrod and Bennet [5]. However, Galam showed that the proposed model had serious flaws [6]. On this basis, Galam constructed a coalition model combining both Edwards and Andersons and Mattis spin glass models to explain the stability in Europe during the Cold War [7]. Latter, Florian and Galam applied the model to describe the fragmentation of former Yugoslavia [8]. More than a decade after, Vinogradova and Galam established The Natural model of coalition in a series of papers [9, 10, 11].

2.2 Mapping social interactions onto spin glasses

Along the Edwards and Andersons spin glass model introduced in Section 1.4.1, each lattice site i represents an actor with spin variable σ_i. The spin variable can take two values: $\sigma_i = +1$ or $\sigma_i = -1$ and represents the coalition the actor is part of. Every actor wants to be in a coalition with its friends and in the opposite coalition of its enemies. Actors interact with one another through the coupling constants that connect their lattice sites, although in the Natural model the coupling constants are generally referred to as 'propensities'. The propensity between two actors in the Natural model is determined by various factors such as historical relations, ethnicity and religion [9],[11]. Two actors are friends if the propensity between them is positive. They are enemies if their mutual propensity is negative. The higher the absolute value of the propensity between two actors, the stronger these feelings of friendship or animosity [10]. Just as in the physical model, these propensities fluctuate on time scales much larger than those in which actors change their spin values [9].

2.3 Spin glasses versus Natural model

There are two key differences between the Edwards and Andersons spin glass and a Natural model system: The individualism of the actors and their rationality when it comes to making a decision on changing spin value. These lead to some differences in dynamics between spin glasses and Natural model systems.

2.3.1 Communist spin glasses and selfish actors

Interactions in a physical Edwards and Andersons spin glass are governed by the minimization of the Hamiltonian of Section 1.4.1. This means that every individual spin in the spin glass acts on what is best for the entire system. This communist, collective spirit is missing in Natural model systems as every actor in a Natural model system is guided by the minimization of its individual Hamiltonian H_i shown in Equation (2.1) [10].

$$H_i(\{\sigma_i\};\{J_{ij}\}) = -\frac{1}{2}\sum_{j}^{N} J_{ij}\sigma_i\sigma_j \tag{2.1}$$

We make a little side step here to introduce the concept of the individual benefit B_i. The equivalent of the minimization of Equation (2.1), is the maximization of Equation (2.2) as this Equation is nothing more than Equation (2.1) where the negative sign in the front is removed. Equation (2.2) is called the individual benefit B_i and is used more frequently in the theory of coalition forming than the individual Hamiltonian. We, too, will be using it instead of the individual Hamiltonian going forward in this work[2].

$$B_i = -H_i = \frac{1}{2}\sum_{j}^{N} J_{ij}\sigma_i\sigma_j \tag{2.2}$$

The maximization of the individual benefit means that an actor in a Natural model system will try to bring the system in a configuration $\Sigma_{max\,i}$, in which this actor achieves its highest benefit $B_{i,\,max} = B_i(\Sigma_{max\,i})$ [9], regardless of the consequences for the system or the other actors, by choosing its spin value based on the individual benefit $B_i(\Sigma)$ that is connected to each system configuration Σ. As an example we will return to the system introduced in Section 1.3. The system can be used in the Natural model as a simplified way to understand the diplomatic squabbling between the great powers of Europe at the brink of the first World War. It is reintroduced in Figure 2.1 but this time with France, Germany, Russia and the United Kingdom as actors on the lattice sites[3]. In Figure 2.2, the different topological diagrams of the system (see Figure 1.8) are shown once again but this time with the individual benefit of every actor shown in its respective corner. In the Natural model, these diagrams refer to different coalition distributions that can be made. Actors connected through a satisfied, positive propensity are members of the same coalition (whether that is the *+1* or the *−1* coalition). Actors connected by a satisfied,

[2] Why introduce the benefit B_i of an actor *i*, one can ask. The reasoning is the following: The benefit *B* can be thought of as money in a bank account. Trying to minimize the taxes you have to pay or trying to maximize the money in your account are equivalent but generally the goal is to increase your money while paying less taxes is a way to achieve this goal.

[3] The propensities can be explained as follows: France and Germany were arch enemies during 19th century. The Russian tsar, in the mean time, was trying to build an empire: Russia was expanding to the west, towards Germany, and to the south, towards India (the 'jewel of the British empire'), souring the relations with Germany and the UK. While the United Kingdom feared this Russian expanse and was sympathetic to the French, it had in general a diplomatic policy of isolationism. This explains the propensity of size zero between the UK and Germany (this propensity is therefore not shown in the Figure). France and Russia also had no meaningful relation as friends or enemies (and this propensity of size zero is not shown). However, note that this is a simplified system, in reality there were more than twenty countries involved in the first World War.

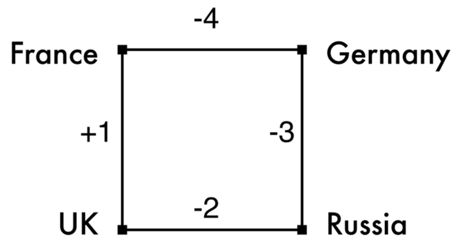

Figure 2.1: The diplomatic ties between the main European powers in the spring of 1914. The propensity between the different actors is shown. If certain actors are not connected with each other, they have a propensity of size zero.

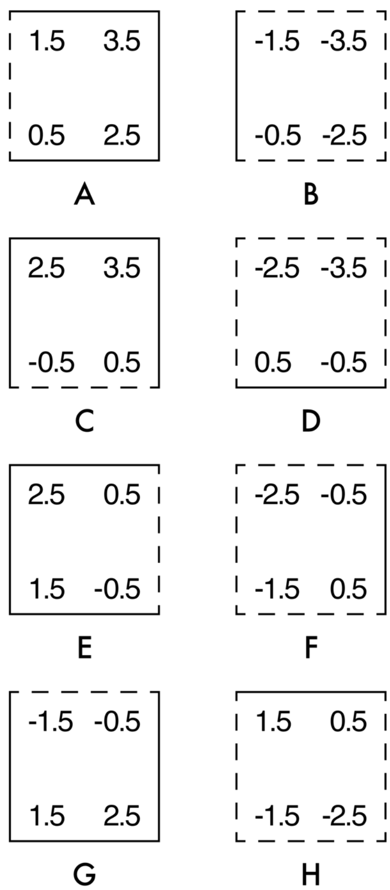

Figure 2.2: The topological diagrams of the system in Figure 2.1. The individual benefit an actor has in a diagram shown next to the actor's lattice site. The diagrams on the left (Diagrams A, C, E, G) are diagrams in which two actors at a time can achieve a maximal benefit.

negative propensity on the other hand, are in different coalitions. The opposite holds for actors connected by unsatisfied propensities. As an example, We can take a look at how history unfolded after the spring of 1914. Germany declared a war on France, followed by one on Russia. The United Kingdom joined the war on the French side. This would put the system in diagram C of Figure 2.2 (Germany versus the others). When Germany and Russia signed a peace treaty. The coalitions changed to those of diagram E

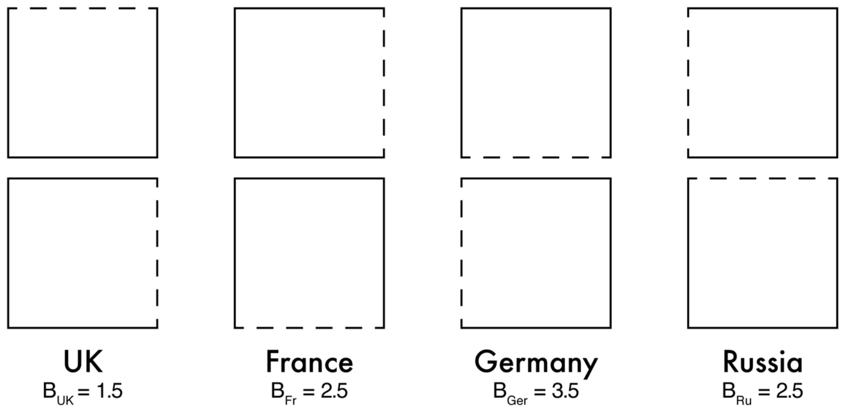

Figure 2.3: The set of diagrams of maximal benefit Γ_i for each actor *i*. Note that every diagram is part of the set of maximal benefit diagrams of two actors and that no diagram is part of all sets.

One can see from Figure 2.2 that there exists a set Γ_i of two diagrams for every actor *i* in which actor *i* has all its propensities satisfied. These two diagrams are connected to the maximal benefit $B_{max\,i}$ for this actor. The four sets of maximal benefit are shown in Figure 2.3. The intersection of these sets of maximal benefit Γ_{Fr}, Γ_{Uk}, Γ_{Ge} and Γ_{Ru} is the empty set and thus, there is no system configuration in which all actors achieve their maximal benefit. This is in contrast to the example from Section 1.3 where the system as a whole preferred a single diagram (diagram A in Figure 1.8).

2.3.2 Short sighted spin glasses and rational actors

The other key difference between spin glasses and actors in Natural model systems is how they go about achieving their goal (minimizing the system energy and maximizing their individual benefit respectively). Spins try to minimize the system energy via a gradient descent approach. In this process, a spin knows the energy level of the current system energy and it knows the change in energy if it were to invert its spin value. If a change of spin value lowers the system energy, it will accept the new value. All spins in the system check regularly if they should change their spin value. When no more spin changes occur, the system is in an energy minimum. This is however, where the gradient descent approach runs into trouble. It is possible that this energy minimum is a local energy minimum. The spins do not find any way to lower the system energy and the system does not attain its global energy minimum configuration [12].

Actors in a Natural model system, on the other hand, consist of human beings. They are capable of rational behaviour[4]. This means that an actor grounds its decision on whether to change its spin value and coalition on its ability to forecast future system configurations [9]. This forecasting of future system configurations depends on the knowledge of the actor about the set of propensities $\{J_{ij}\}$ [10]. With this knowledge, the actor is able to calculate the (maximal) benefits of every actor in the current system configuration and in possible future system configurations. This allows the actor to predict how the other actors will react on the current configuration [9], taking into account that they also want to maximize their benefit, and thus how the system will evolve. The actor now chooses a spin value based on which option (keeping its current value or inverting it), combined with the other actors' reactions, brings the system closer to a system configuration in which it achieves its maximal benefit [10]. This kind of reasoning based on forecasting is called extended rationality [10] and an actor possessing it, is able to overcome local benefit plateaus because it knows its maximum benefit value through its knowledge of propensities. As long as the actor is not yet in a configuration in which it achieves its maximal benefit, it will keep looking for a way to achieve it. The actor is also able to settle for a temporary lower benefit if it knows that the suboptimal configuration attached to it leads to a configuration attached to its maximal benefit in a later step

[4] They are also capable of irrational behaviour: They hold grudges and are capable of taking revenge. These dynamics are discussed in the following Chapter.

[10].

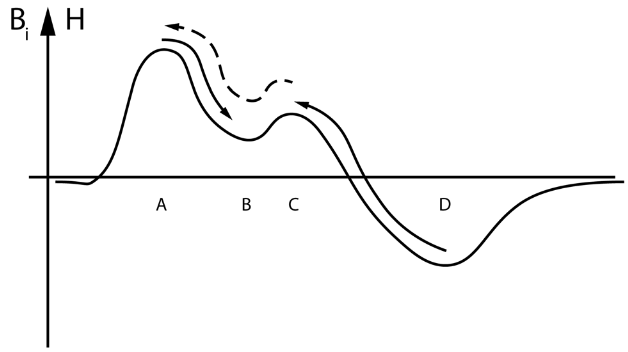

Figure 2.4: An example of both an energy and benefit curve. Individual spins lower the system energy by moving the system from point A to B. However, the system is stuck in local energy minimum B because the spins do not find a way to lower the energy to absolute minimum D without raising the energy first. An actor in a Natural model system, on the other hand, starts in point D and tries to reach point A. Limited actors are stuck in point C and only extended actors can reach point A.

Of course, some actors have more information about the system (the propensities) than others. (Think of the United States with all its three letter intelligence agencies). To make the distinction between these kind of actors and other actors with a more limited knowledge about the system and its propensities, the concept of limited rationality is introduced [10]. Because of their lack of information, actors possessing limited rationality are not able to adequately calculate the benefits in the system and, as such, do not know their true maximal benefit. They have to resort to a strategy of immediate gains [10, 11]. The approach is similar to the gradient descent method of the physical spins. They invert their spin value if this raises their benefit. If it does not, they keep their current spin value and the coalition attached to it. Actors possessing limited rationality cannot overcome local benefit plateaus, the same way physical spins cannot overcome local energy minima.

2.4 Frustration in the Natural Model

The dynamics of the Natural model discussed in the previous sections have consequences for the concept of frustration in the Natural model as compared to the concept in a physical sense. Remember the example of Section 1.3 (Figure 1.6), the system was not frustrated even though the product of all the coupling constants was negative. When this system was reintroduced in Section 2.3.1 as a system of actors with extended rationality, every actor tried to achieve its maximal benefit. Because there is no common configuration of maximal benefit, there will always be at least two actors searching for configurations in which they achieve maximal benefit and the system fluctuates continuously between the different configurations of maximal benefit seen in Figure 2.3. Although the system is not frustrated in the physical meaning, it is frustrated in a Natural model sense because of the individualism of the actors and their extended rationality. Such Natural model systems, in which no common configuration of maximal benefit exists, are called unstable. This is the Natural model equivalent of frustration.
So, in a stable system, there exists a common configuration Σ_0 of maximal benefit. This requires that all the propensities are satisfied in this configuration Σ_0. Such a requirement for a stable Natural model systems leads to a topology show in Figure 2.5. This hypothetical topology diagram is equivalent to a unique ground state in the physical spin glass model and brings us to the following conclusion: The necessary frustration condition of Chapter 1 (Equation (1.2)), is a sufficient one in Natural model systems with only extended actors [10, 9]. This means unstable Natural model systems with only extended actors are unstable forever. All actors are able to identify the configuration connected to their maximal benefit and will try to bring the system to their preferred configuration [10, 11]. However, if a system consists of actors with extended rationality as well as actors with limited rationality, the instability can be stopped by trapping the actors with limited rationality in a local benefit plateau [11]. Since limited actors can be trapped in a system configuration in which they can see no further raise in benefit, the actors with extended rationality can use this to steer the system towards a configuration in which

they achieve their maximal benefit while the actors with limited rationality are stuck with a sub-maximal benefit

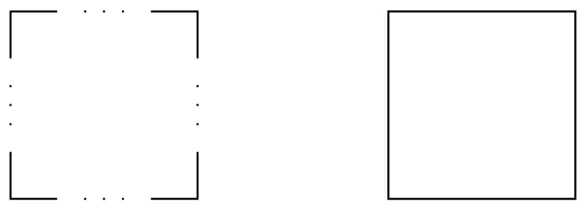

Figure 2.5: The left side of this figure shows the topological requirement for a stable system configuration in a system with extended actors only. This leads to the topological diagram on the right hand side. If such a diagram is topologically not possible, no stable system configuration exists in a system with extended actors only. The system can still be stabilized if there are also limited actors in the system in this case.

[10]. Because the limited actor cannot see any further raise of their benefits, they do not make any further moves and the extended actors get to keep their maximal benefit. On the other hand, if there does exist a single configuration in the system in which every actor achieves its maximal benefit then this state will be reached very quickly in a system with actors with extended rationality. These actors can even help overcome potential benefit plateaus towards this configuration [10]. Natural model systems with a configuration in which every actor either achieves its maximal benefit or no longer sees room for further gains is called a stable system. The process of moving from some unstable configuration towards a stable one is called stabilisation.

Chapter 3 Extensions to the Natural model

3.1 Introduction to Natural model extensions

In the previous chapter, the basic Natural model of coalition forming was introduced and its dynamics were discussed. This model uses interactions based on historical propensities. These propensities are predominantly influenced by the history of conflicts between actors and as a consequence, have a tendency to be negative. This makes balancing such a system very difficult. However, with the defeat of Germany, Japan and Italy in World War II came the rise of communism and the introduction of interactions between countries based on rational principles. This dynamic was established by Vinogradova and Galam in [9] and discussed in depth in the following section. Finally, two other extensions, one introducing diplomatic interactions and another one introducing risky/populist behaviour, to the Natural model of coalition forming are introduced and briefly discussed.

3.2 Global alliance model

3.2.1 Introduction to the global alliance model

The best way to introduce the concept of global principles and their effect on the formation of coalitions is with a reference to the Cold War. From 1947 to 1991, most countries around the world rallied to, or supported, one of two coalitions. These two coalitions, the Western Bloc and the Eastern Bloc were founded on, among other things, two opposing stances on economics. The Western Bloc favoured free markets and capitalism while the Eastern Bloc preferred a state driven market and communism. Countries primarily chose the coalition they were joining based on their own view regarding economics. The historical ties/propensities between countries did not play a major role in this decision. The dynamic where actors make the decision on whether to join a coalition based on a principle instead of the historical ties with the other actors is not present in the Natural model of coalition forming. To introduce it, we will superimpose the Mattis spin glass model from Section 1.4.2 on top of the Edwards and Andersons spin glass model that forms the Natural model of coalition forming. This extended model is called the global alliance model of coalition forming or GAM [9] and was established by Vinogardova and Galam in [9] and [11].

3.2.2 Mapping global principles onto a Mattis spin glass

The Mattis spin glass model from Section 1.4.2 is used to introduce a global principle G in the Natural model of coalition forming. The actors are now guided by the minimization of the individual benefit

$$B_i = \frac{1}{2} \sum_{j}^{N} (J_{ij} + G_{ij}) \sigma_i \sigma_j, \tag{3.1}$$

where G_{ij} is the propensity due to the global principle [9]]. Besides the spin variable σ, every actor has a belonging parameter ε which can take the values $\varepsilon = +1$ or $\varepsilon = -1$ and represents the actor's stance on global principle G. It can be understood as the spin value the actor would like to have because of this global principle [9]. For example, an actor i with a belonging parameter $\varepsilon_i = +1$ favours the global principle G and wishes to be part of a coalition with other actors whom also support it. An actor j with a belonging parameter of $\varepsilon = -1$, on the other hand, is opposed to the global principle G and will join a coalition with other actors whom oppose it. Spin variable σ still represents the actual coalition an actor is part of [9]. The definition of propensity G_{ij} between actors i and j because of the global principle G is shown in Equation (3.2). The sign is the product of respective belonging parameters ε_i and ε_j while the size of the propensity is a function of the effect R_{ij} of global principle G on these two actors[5]. The function should be defined in such a way that $G(R_{ij}) > 0$, meaning that the global principle is intrinsically beneficial to the actors. When two actors have the same value for belonging parameter ε, their mutual propensity due to the global principle G_{ij} is positive. Aligning their spin values as well will increase the individual benefit of both actors and they end up in the same coalition. However, if the actors have different belonging parameters, their global principle propensity will be negative and their individual benefit can only be increased by choosing differing spin values: They end up in different coalitions.

$$G_{ij} = G(R_{ij}) \epsilon_i \epsilon_j \tag{3.2}$$

The belonging parameter ε and spin variable σ do not need to be aligned but when the global principle propensity is larger in size than the historical propensities it is, looking at Equations (3.1) and (3.2), more beneficial for an actor if they are. We connect a note on symmetry to this statement: The spin inversion-symmetry is not broken by introduction of belonging parameter ε. All actors aligning their spin variable and their belonging parameter yields the same result as all actors anti-aligning their spin variable and belonging parameter.

3.2.3 Dynamics of the global alliance model

To study the dynamics of the global alliance model, we are now going to take a look at a system where the effect of a global principle G on all actors is of such a scale that the historical propensities between actors can be neglected with respect to the size of the global principle propensities. The system is shown in Figure 3.1 and is a simplified version of the Balkan region before World War I. The main actors in this region at the time were Bulgaria, Greece, the Ottoman Empire, Romania and Serbia. During the 19th and early 20th century, there were many conflicts in which these countries were constantly switching alliances. Due to the conflict-ridden mutual history, we assume that all the historical propensities J in the system are negative: $J = -1$, meaning that each country loathes the other four equally.

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5 This effect R_{ij} is equivalent to the distance function between two neighbours in the Mattis model (see Section 1.4.2).

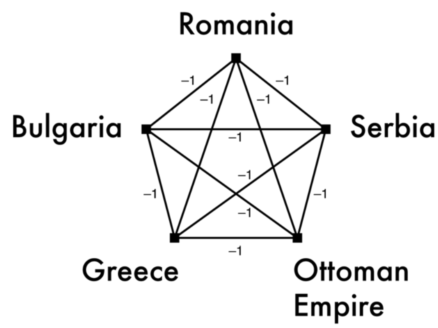


Figure 3.1: The Balkan system before World War I. All countries hate each other equally and the system is unstable as a consequence.

This latter assumption implies that all actors possess extended rationality and therefore are able to identify their maximal benefit. When checking the stability condition from Chapter 2 and Chapter 1 (Equation (1.2)), we can conclude that the basic system is unstable. For completeness, the topological diagrams are shown in Figure 3.4 and indeed, no topological diagram where all propensities are satisfied, exists. We fast-forward the system to 1947. Greece and Turkey (the successor of the Ottoman Empire) just joined the NATO while Bulgaria, Serbia and Romania were communist satellite states. The NATO and the USSR gave their members military support and protection through the 'An attack against one is an attack against all'-principle. This military support had a major impact on political decision's and relations between countries. For example, Greece and Turkey joined the same coalition even though they were enemies for most of history. This means that we can interpret the military support and the protection both coalitions offered as a global principle on which countries had to choose a side.

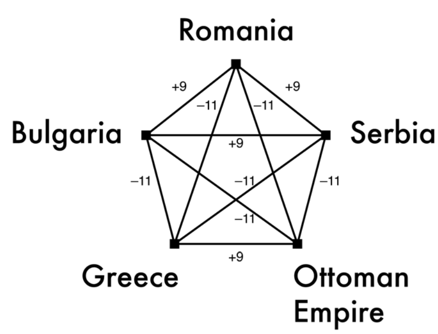


Figure 3.2: The Balkan system during the Cold War. Greece and Turkey (the successor of the Ottoman Empire) had a belonging parameter favouring the global alliance of NATO while the other three countries had a belonging parameter preferring the USSR. Because the propensities due to the global principle of economy trump the historical propensities, the system is able to stabilize.

In the model, we identify the NATO coalition with the positive spin value and belonging parameter while the USSR is represented by the negative spin value and belonging parameter. This means that both Greece and Turkey have a positive belonging parameter. Bulgaria, Serbia and Romania have a negative belonging parameter. We will further assume that the propensity because of the global principle trumps the historical propensities and is equal in size for every pair of countries in the system: $G_{ij} = 10$. The effect of the global principle on the system is shown in Figure 3.2. Now there exists a stable system configuration because the stability condition holds for every cycle in the system. The stable configuration includes a coalition of Greece and Turkey and another coalition containing Bulgaria, Serbia and Romania. This example shows that, while it is not trivial to stabilize a Natural model system based on historical propensities, it is relatively easier to stabilize a global alliance system

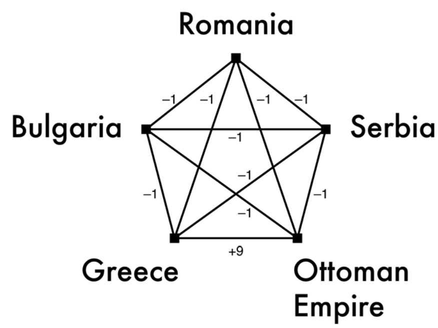


Figure 3.3: The Balkan system after the collapse of the USSR. Greece and Turkey still belong to the NATO and keep interacting with each other based on the global principle propensity. The other countries return to interactions based on historical propensities, creating an unstable subsystem.

if the effect of the global alliances on the actors has such a size that the historical propensities can be neglected. However, when the importance of the global principle to the actors decreases, there is a return to the dynamics based on the historical propensities. For an example of this, we can fast-forward our Balkan system again to 1991, the year when the Soviet Union collapsed. The dissolution of the USSR can be represented by the belonging parameters for Bulgaria, Serbia and Romania suddenly becoming zero as they no longer have an affinity to belong to the Eastern Bloc. The belonging parameters for countries which are part of the Western Bloc remain unchanged as the NATO continued to exist even after the dissolution of the USSR. Since the former communist countries are no longer guided by a global principle, they interact with each other based on the historical propensities (which are still $J = -1$). The NATO members Greece and Turkey still interact with each other based on the global principle propensity. At the same time, they are become limited actors because their membership of NATO allows them to have a global perspective, in which their membership and the security it offers is more important than finding their maximal benefit. In the same context do the spin values in the cycle of former communist countries not represent membership of a global alliance, they represent local coalitions. The effect of the dissolution of the Soviet Union is shown in Figure 3.3. It is clear that the cycle containing the former communist countries is no longer stable. The alliance between Greece and Turkey remains stable because they still reap the benefits of the support and protection of the NATO (and the historical propensities are still negligible to them). Greece and Turkey's spin values will therefore remain unchanged. The other three countries will enter an endless process of changing alliances in hopes of trying to achieve their maximal benefit. These dynamics of both the NATO members and the former communist countries could also be witnessed in real life events in the Balkan after the Cold War.

3.2.4 Multiple global principles

Taking into account only one global principle can be too restricting. Sometimes there are multiple principles such as economics, moral reasons and politics playing a role in the system [9]. Actor i has an independent belonging parameter $\varepsilon_{i,\alpha} = \pm 1$ for each of the different global principles $\{\varepsilon_\alpha\}$, meaning it can have a belonging to a coalition on one global principle but a belonging to the other coalition on other global principles. Multi-global principle systems are in general, harder to stabilize as the global principle propensities can cancel each other due to the independent belonging parameters for each global principle. In Chapter 4, we will study the stabilisation requirements for multi-global principle systems.

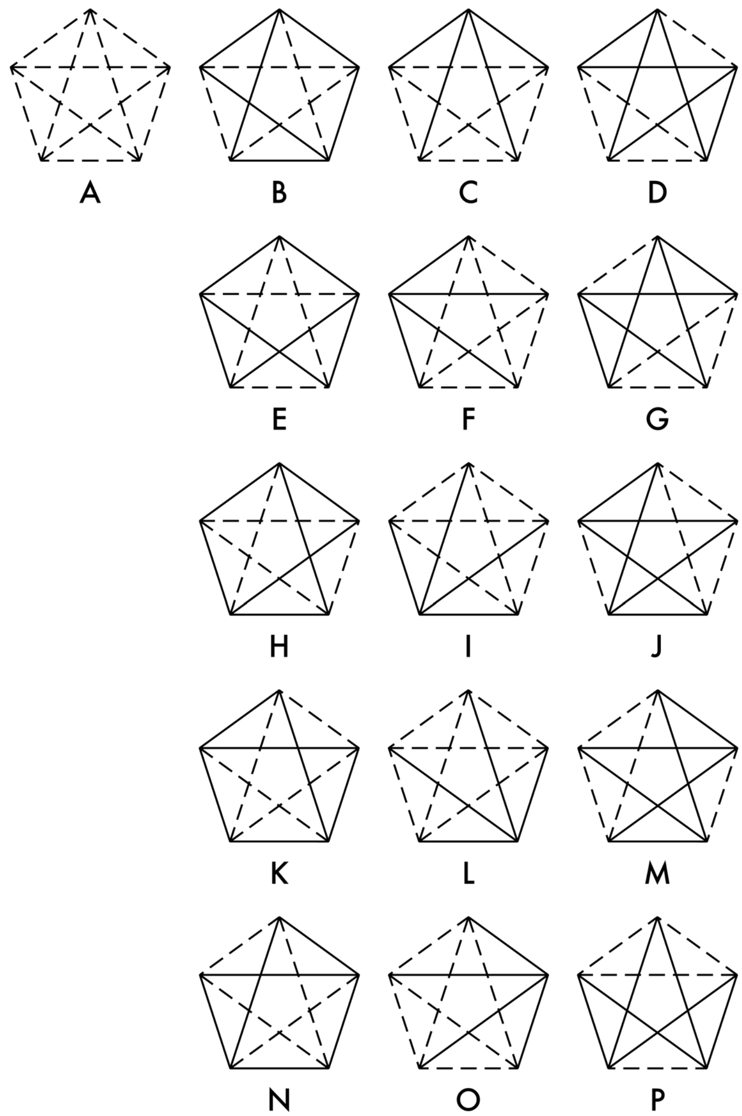


Figure 3.4: The topological diagrams of the Balkan system pre-World War I. If all countries choose the same spin value, they end up in Diagram A. Diagrams C, F, I, L, O represent a situation in which one country is pitted against the other four. The other Diagrams represent the various situations in which two countries form an alliance against the other three.

3.3 Diplomacy

3.3.1 Introduction to diplomatic interactions

When trying to model geopolitical situations, it might be interesting to include diplomatic interactions into the model. In such geopolitical systems, diplomacy is used by an actor to convince others to change their spin value and join its coalition in order to raise its own benefit. In this chapter, the term 'Diplomacy' can take various meanings. It could be the nuclear weapons with which North Korea threatens its neighbours but also the bailout program the ECB (European Central Bank) used to convince Greece not to leave it. The diplomatic interactions can be added to the Natural model of coalition forming (or the GAM) by introducing local magnetic fields at the lattice sites of actors who are being pressured.

3.3.2 Mapping diplomacy dynamics onto magnetic fields

Diplomatic pressure, used by actor *j* to convince another actor *i* to join its coalition, can be introduced in the Natural model of coalition forming by adding a local magnetic field, generated by actor *j* to the lattice site of the actor *i*. This latter actor's benefit becomes

$$B_i = \frac{1}{2}\sum_{j}^{N} J_{ij}\sigma_i\sigma_j + b_i\sigma_i, \tag{3.3}$$

where J_{ij} is the historical propensity between actor *i* and actor *j* and b_i is the local magnetic field on actor *i*'s lattice site generated by other actors. The local magnetic field is a positive addition to benefit B_i if actor *i* decides to comply with it (and thus align its spin value with the magnetic field) while it lowers B_i if the actor decides not to comply and keep its current spin value.

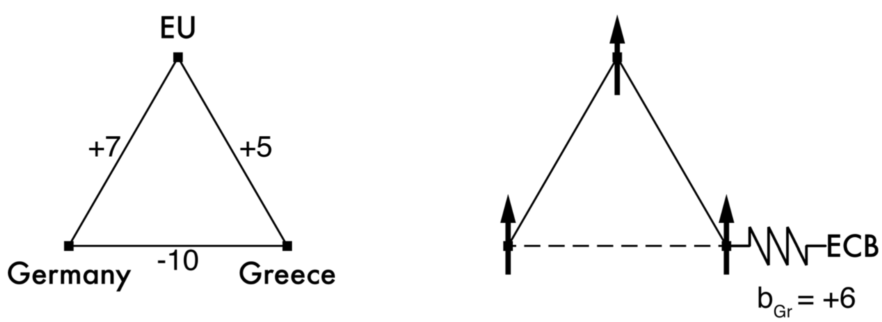


Figure 3.5: The Grexit system (left) contains the most important actors of the crisis: Greece, The EU and Germany as a primary driver behind the austerity measures. One can see that it is more beneficial for Germany and Greece if Greece decides to leave the Eurozone ($B_{GR} = -5$ to $B_{GR} = 5$). However, the EU's benefit is lowered ($B_{EU} = 12$ to $B_{EU} = 2$) in this scenario. It used the ECB bailout program as a local magnetic field to convince Greece to stay in the Eurozone. ($B_{Gr} = +1$ if it stays instead of $B_{Gr} = -1$ if it leaves)

As an example, we take a look at the European Union (EU) in 2012. The Eurozone was gripped by the Grexit crisis and tensions were running high between the Greek and German government. Germany was one of the countries who insisted on strict austerity measures in return for a bailout program for Greece by the European Central Bank (ECB). Such was the hostility in Greece towards Germany that the country reopened claims to World War II reparations [19]. This highly complex, example can be simplified into the system shown in Figure 3.5. The system consists of Germany, Greece and the institution of the European Union, the main actors in the crisis. The EU can be thought of as an extended actor because of its many connections to the other European countries as well as international institutions such as the International Monetary Fund (IMF) while Germany and Greece are limited actors. A positive spin value in the system represents a membership of the Eurozone while a negative one represents being out of the Eurozone. As such, all three actors start with a spin value of $\sigma = +1$. In terms of the propensities, we assume the following: The propensity between Germany and Greece is negative and is a function of the animosity between both countries because of the Greek debt to Germany, meanwhile the propensity between a country and the EU is positive and proportional to country's preference to be a part of the EU. It is safe to assume that, at the height of the crisis, the animosity between Germany and Greece was stronger than Greece's desire to be a part of the EU [20]. So why did Greece not leave the Eurozone? Since a Grexit would lower the benefit of the EU, it is possible that the bailout program of the ECB acted as a local magnetic field at Greece's lattice site generated by the EU in order to pressure Greece to stay in the Eurozone.

This example also shows how the local magnetic fields break the spin inversion symmetry. There is a difference between all the actors having a spin value of $\sigma = +1$ (being in the Eurozone) and all the actors having a spin value of $\sigma = -1$ (the total collapse of the Eurozone).

3.4 Risky actors

3.4.1 Introduction to irrational behaviour

Sometimes actors take decisions that are not based on rational arguments. Instead, decisions are based on the arguments of religion, electoral gain or populism. In Chapter 2, we attributed actors with a lack of knowledge about the system with limited rationality. It is however, possible that an actor has all the knowledge about a system but decides not to act on it. One such an example might be the case of Brazil's COVID-19 response. Its president, Jair Bolsonaro, minimized the impact the virus would have on the country and did not order a lockdown of the country, even when health experts recommended it as an important measure to stop the spread of the virus [18]. This kind of risky behaviour can be introduced in the Natural model of coalition forming by introducing the concept of local temperature.

3.4.2 Mapping risky behaviour onto temperature

Risky behaviour from an actor *i* with limited rationality can be accounted for by introducing a local temperature T_i at its lattice site. When making a decision in normal circumstances, that is $T_i = 0$, actor *i* would only accept a change in spin value if this would raise its benefit. However, when $T_i > 0$, the probability of actor i accepting a change in spin value if this does not raise its benefit is $exp(-\Delta/T_i)$, where $\Delta = B_{i,\, current} - B_{i,\, new}$ is the difference between the current benefit and the new benefit. This means that for a given local temperature the lower the benefit of the new spin value, the lower the probability that this unfavourable change will be accepted. When a change in spin value raises the benefit, it is still accepted with a probability of 1. This also holds for actors possessing extended rationality. When they do not find their maximal benefit in the first step of their rationality tree, there is a probability of $1-exp(-\Delta/T_i)$ that they keep on looking for their maximal benefit in further steps of their rationality tree and a probability of $exp(-\Delta/T_i)$ that they make a rash decision and change their spin value immediately. This new dynamic increases the randomness in the system and can help limited actors overcome local benefit plateaus. It also makes it more difficult for actors possessing extended rationality to map a path towards their maximal benefit.

PART II SIMULATIONS

Chapter 4 Implementation of the Natural model of coalition forming

4.1 Introduction and computational setup

In this chapter, we discuss how we implemented the Natural model of coalition forming and its extensions as described in the previous chapters. The scripts for the computational parts of the simulations, have been coded in Python, making extensive use of its libraries. Python is a high-level interpreted general-purpose programming language, that is widely used thanks to its simplicity and readability. The numerical calculations have been done using the library NumPy that contains many functions written in C/C++, speeding up the calculations [13, 14]. Calculation time was also minimized with the use of the built-in Python library Itertools. The visualisations made in the development phase for better comprehension as well as those that visualize the simulation data are coded in javascript [16] and specifically D3.js, a JavaScript library for producing dynamic, interactive data visualizations in web browsers [15]. In Chapter 5, we discuss the results of the simulations.

4.2 Implementation of rationality

The key feature of the natural model to implement in a simulation program is the concept of rationality as actors use it to make a decision on whether to change their spin value, and hence their coalition. Before we explain how we went about implementing the concepts of limited and extended rationality, we address an important question first: Do the actors in the system make their move (keep their current spin value or not) all at the same time, based on the same system configuration or do they make their moves sequentially, based on the system configuration where other actors have already updated their spin value. We opted for the latter algorithm because it gives the first actor to make a decision a big influence over the future system configuration the current configuration will evolve into. To see this, remember that the system has 2^N possible system configurations. Before the first actor makes his move, these can still all be reached. When the first actor makes its move, the number of possible future system configurations the current can evolve into gets halved. Each time another actor makes a move, the number of possible future system configurations gets halved. When the last actor makes its move, it can only

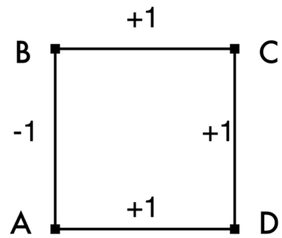


Figure 4.1: An example system to illustrate the implementations of the various rationality trees. We let actor A be an extended actor while the other actors B, C and D possess limited rationality. This order of decision taking is therefore: A-B-C-D. The system starts with all actors having the same positive spin value $\sigma = +1$.

choose from two system configurations. Thus, the last actor has very little sway over the system. This power asymmetry is something we associated in previous sections with the difference between actors who possess extended rationality and those who possess limited rationality. When doing simulations with sequentially updated natural model systems, we noticed that additional power is given to the actor with the highest value of maximal benefit. To understand this, note that this actor, because of his high maximal benefit, must be connected to other actors with high propensity values. When other actors try to increase their benefit, they will above all try to satisfy the propensities with the highest propensity values. These propensities most likely connect them to the actor with the highest maximal benefit. For these reasons, we define an actor with extended rationality as an actor with a high maximal benefit. This actor will also be the first one to make his move on changing its spin value or not.

4.2.1 Limited rationality

Actors with limited rationality try to maximize their immediate benefit. To do this, limited actor *i* compares its benefit $B_i(\Sigma_{current})$ attached to the current configuration $\Sigma_{current} = \{\sigma_1, ..., \sigma_i, ..., \sigma_N\}$ to the benefit $B_i(\Sigma_{flipped})$ of the configuration $\Sigma_{flipped} = \{\sigma_1, ..., -\sigma_i, ..., \sigma_N\}$ where it has inverted its spin value. Actor *i* chooses the spin value associated with the highest benefit of the two. If the benefits of the two configurations are equal to each other, the actor inverts its spin value with a probability of 50 percent. This can be visualized in an outwards rooted tree[1] which we call the 'rationality tree'. The starting node of the tree is the current configuration. The children of this node, the decision nodes, are the current configuration and the one where actor *i* inverted its spin value. A rationality tree for limited actor *B* of the example system in Figure 4.1 is shown on the right hand side of Figure 4.2.

4.2.2 Extended rationality

An actor with extended rationality does not only get to make the first move, it also possesses the ability to forecast how the other actors will react on its move in their turn. For this, the actor with extended rationality, hereafter referred to as an extended actor, also builds a rationality tree. The starting node for the tree is, like in the limited rationality case, the current configuration. The children nodes are also the two system configurations associated with the extended actor either keeping its current spin value or inverting it. The extended actor, knowing that the limited actors try to improve their immediate benefit, can now forecast how they will react on each of the two options. These reactions on a child node transform it in a decision node. If the extended actor achieves its maximal benefit in one of the decision nodes, it chooses the spin value associated with this decision node.

[1] An outwards rooted tree is a tree in which one vertex has been designated the root. The other edges of the tree have an orientation away from the root. For an outwards rooted tree in which vertex u is called the root and containing another vertex v, there is exactly one directed path from u to v. [17]

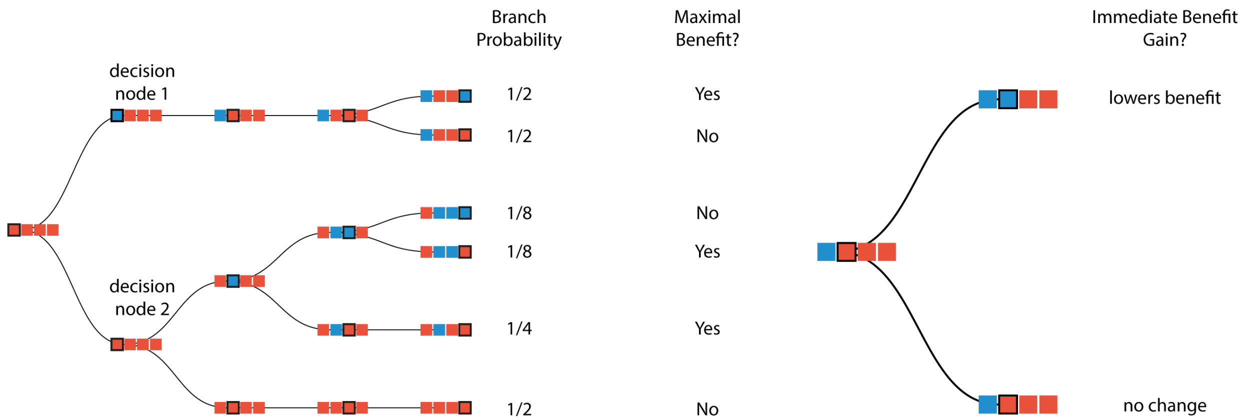


Figure 4.2: The rationality trees for the extended actor A (on the left hand side) and for the limited actor B (on the right hand side). All actors in the system start with a spin value of *+1,* represented by a red rectangle. A blue rectangle represents an actor with a spin value of *−1*. In the rationality tree of actor A are the actors who take a decision represented by a black outline. When an actor (other than A) encounters a degeneracy in this tree building process, it splits the branch in two.

If both the decision nodes do not hold the maximal benefit of the extended actor, each decision node is used as a starting node for a new round of tree building until a decision node with the maximal benefit of the extended actor is found. An example of an extended rationality tree for extended actor A of the system of Figure 4.1 is shown in Figure 4.2. The attentive reader can now raise two questions. First: 'What happens when a limited actor encounters a degeneracy in the tree building process when reacting on a child node?' The rationality tree takes both options into account by making a separate branch of the child node for each option of the limited actor. A probability of fifty percent is attached to each branch. Every additional degeneracy in the tree building process divides a branch in two new branches and the probability of each of the new branches is halved. At the end of the process, the extended actor chooses the decision node where it achieves a maximal benefit with the highest probability. Second: 'What happens when there are multiple extended actors in the system?' In this case, the tree building process of one extended actor i would have to take into account the decision process of another extended actor j. In this decision process inside the decision making process of actor i, extended actor j takes into account the decision process of extended actor i as well and so the infinite cycle continues. To reduce the recursive nature of such a situation, extended actors treat each other as limited actors in their tree building.

4.3 Implementation of diplomacy

Some actors i have the possibility to pressure another actor j into making a move to improve their own benefit B_i. This kind of dynamic was discussed in Section 3.3. It is implemented by giving the diplomatic actor i the option to create a local magnetic field at the lattice site of victim actor, j who can help the diplomatic actor to improve its benefit by inverting spin value. The next time the victim actor has to make a move, this local magnetic field is taken into account in calculating its benefit. The local magnetic field is subsequently reset to zero after the victim actor has made its decision.

Chapter 5 Case studies on the implementation of the Natural model

5.1 Introduction

In order to test whether the dynamics of the Natural model discussed in Chapter 2 and the dynamics of the Global alliance model discussed in Chapter 3, are present in the simulation program, we will run simulations of the examples presented in these chapters. In these simulations, there are no risky actors or actors who use diplomacy. All case studies follow the same pattern: We start with a simulation in which all the actors are limited actors. We compare this result with the result from the same system with all actors possessing extended rationality and the results from this system where only one actor possesses extended rationality. In a simulation, we let the system take *1000* steps in the configuration space starting from the start configuration. This number of steps is a hyperparameter of the simulation and should be tweaked according to the complexity of the system being simulated. Systems with more actors or more global principles require more steps for the actors to find a stable configuration. Each step creates a new system configuration Σ_{new} based on the previous configuration Σ_{prev} and this new configuration Σ_{new} is formed by the substeps that are the decision making processes of the actors. When the system does not change system configurations ($\Sigma_{new} = \Sigma_{prev}$) for more than *10* steps, the simulation is ended as the system has found a stable configuration.

5.2 Case study 1: Unstable, degenerate system

5.2.1 Case study 1A: All actors possess limited rationality

We start with a simulation of the basic Natural model *4*-cycle system used in Chapter 4 (Figure 4.1). There are no global principles at play and there are no risky actors or actors who use diplomacy. Since the size of all propensities is the same, the maximal benefit of all actors is the same as well. It can be checked that the maximal

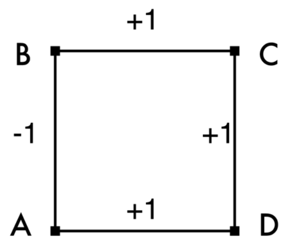


Figure 5.1: The system used in Case studies 1A, 1B and 1C. Actor A is an extended actor while the actors B, C and D possess limited rationality. This order of decision taking is: A-B-C-D. The system starts with all actors having the same positive spin value $\sigma = +1$.

benefit of an actor is $B = 1$. However, the limited actors are not aware of this. The diagrams in which every actor achieves its maximal benefit are shown in Figure 5.2. After running the simulation, one can see in Figure 5.3 that the actors were not able to find a stable system configuration and all actors endlessly switch coalitions in order to achieve their maximal benefit. Figure 5.3 shows the number of simulation steps a certain topological diagram was created by the actors decisions. One sees that, in this case, the system spends the same amount of 'simulation time' (i.e. the number of steps the actors created a certain diagram) in the four topological diagrams of Figure 5.2. This means that all actors succeed in achieving their maximal benefit, although only for a few steps before another actor brings the system in another configuration by switching from coalition in order to raise its own benefit. Note that the 'excited' diagrams (i.e. those with more than one unsatisfied coupling) are not visited once during the simulation.

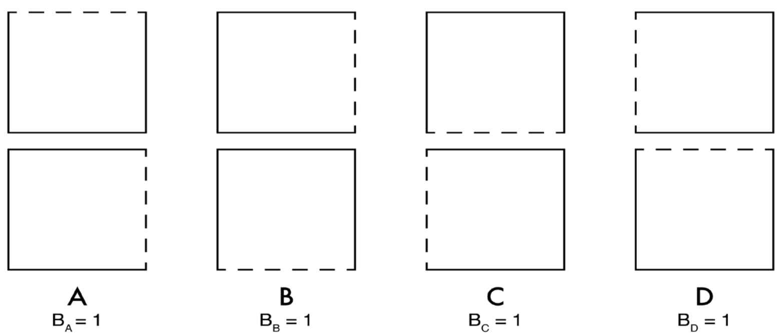


Figure 5.2: The topological diagrams associated with a maximal benefit for the actors of the system shown in Figure 5.1.

5.2.2 Case study 1B: All actors possess extended rationality

In contrast to the previous case study, all actors possess extended rationality and know the value of their maximal benefit ($B = 1$). During the simulation, the system fluctuates between only two topological diagrams, as can be seen in Figure 5.4. These two topological diagrams can be associated with the maximal benefits of two actors at a time (see Figure 5.2). It is clear that the system spends the same amount of simulation time in both topological diagrams and the second actor continuously achieves its maximal benefit while the first and third actors, actors A and C achieve, alternately, their maximal benefit. Actor D is not able to achieve its maximal benefit. Before the substep of this actor's decision making process, the other actors have, in their substeps, topologically already eliminated the option of actor D achieving its maximal benefit. The reason actor B is able to endlessly enjoy a maximal benefit is because it has no connection to the last actor, actor D. Actors A, B and C choose a coalition based on their forecast of a future maximal benefit. However, the maximal benefit of actor B is only dependent on its connections to actors A and C, while their maximal benefit is partly dependent on their connection to actor D. While trying to maximize their benefit actors A, B and C satisfy their connections with each other and thus give actor B its maximal benefit. Actors A and C each still depend on actor D satisfying its propensity with them in order to achieve their maximal benefit.

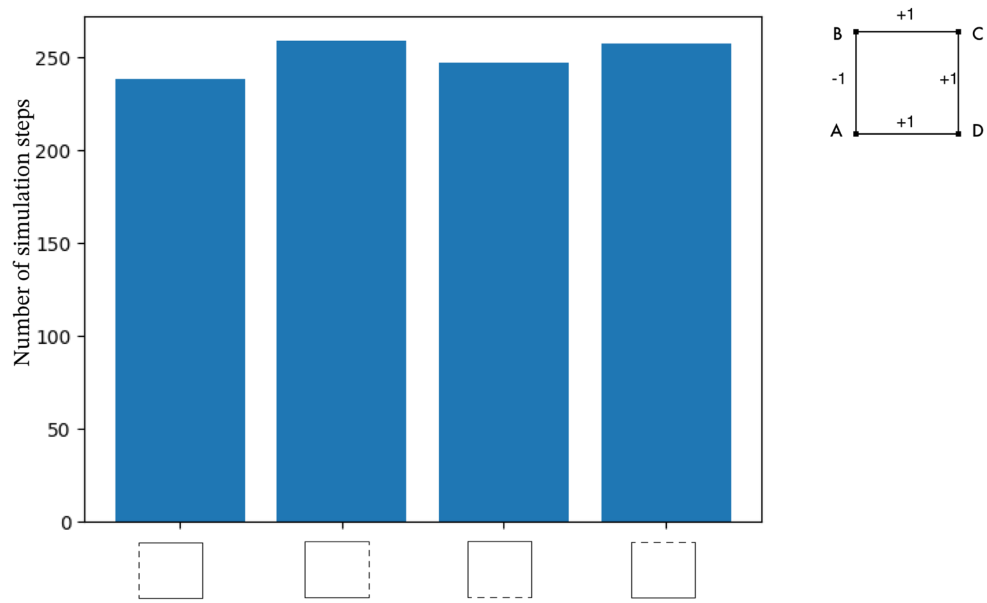


Figure 5.3: The topological diagrams of the system of Case study 1A that were created during a simulation of *1000* steps. The vertical axis shows the number of steps that resulted in the topological diagrams on the horizontal axis. All actors have an equal opportunity at achieving their maximal benefit.

In actor D's decision process, however, it can only choose out of the two system configurations from Figure 5.4, neither of which are associated with or lead to a maximal benefit for this actor. Since these system configurations have the same benefit to actor D, it chooses one of them with a probability of *50* percent. Its decision does grant either actor A or C a maximal benefit. This simulation shows the lack of power the last actor in the decision making process has in a system containing actors with extended rationality.

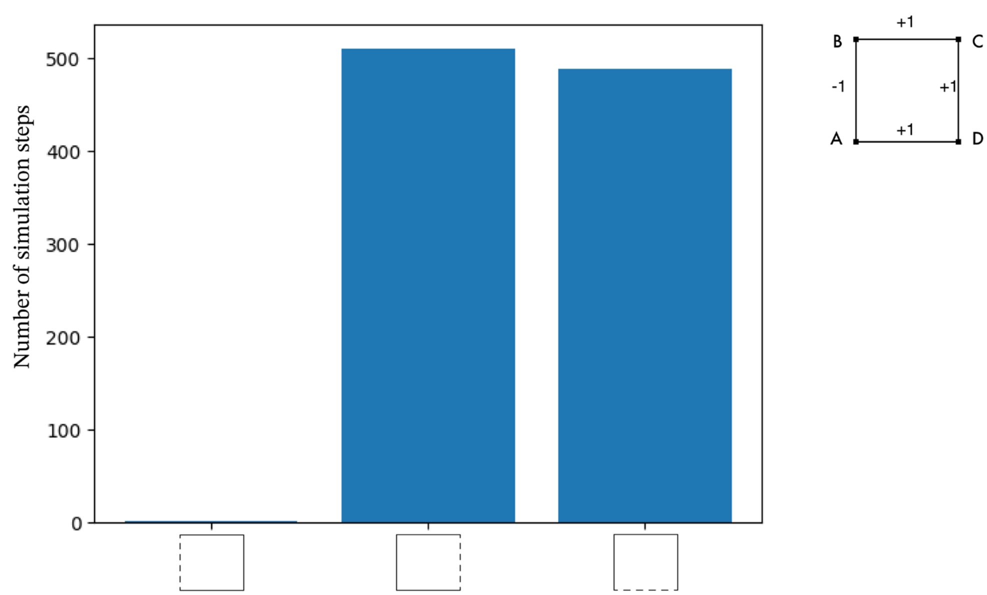
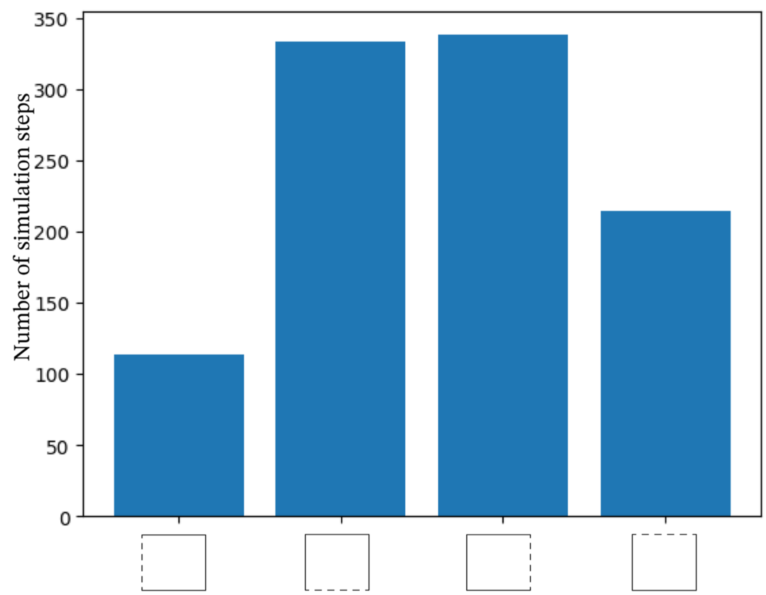


Figure 5.4: The topological diagrams of the system of Case study 1B that were created during a simulation of *1000* steps. The vertical axis shows how many steps resulted in the creation of the topological diagrams shown on the horizontal axis. The diagram on the left is the start configuration and is never visited again. The system moves back and forth between two diagrams in which the first three extended actors are able to achieve a maximal benefit. The last actor does not achieve its maximal benefit.



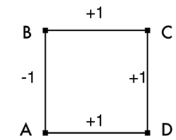


Figure 5.5: The topological diagrams of the system of Case study 1C that were created during a simulation of *1000* steps. The vertical axis shows how many steps resulted in the creation of the topological diagrams shown on the horizontal axis. The first and only extended actor is in the majority of the simulation steps able to steer the system towards system configurations in which it achieves its maximal benefit. This in contrast to Case study 1A where the actors all had an equal opportunity to achieve their maximal benefit but also in contrast to Case study 1B where the last actor was not able to achieve its maximal benefit.

5.2.3 Case study 1C: Only the first actor possesses extended rationality

Now, we study the effect of a single extended actor in the system. Actor A possesses extended rationality and is also the first one in the decision making process. Note that the rationality trees of the first two actors for this case were shown in Chapter 4 (Figure 4.2). As can be seen from a comparison between Figure 5.3, Figure 5.4 and Figure 5.5, this case study is the transition case between the previous two case studies: The system still visits the same four topological diagrams as in Case study 1A and, unlike Case study 1B where the last actor was unable to reach its maximal benefit, every actor is able to achieve its maximal benefit. This time, similar to Case study 1B, the extended actor can leverage its extended rationality to skew the distribution of configurations visited towards system configurations in which it achieves its maximal benefit.

5.3 Case study 2: Unstable, non-degenerate system

5.3.1 Case study 2A: All actors possess limited rationality

To study the effect of the size difference of the propensities between actors, we use the World War I system introduced in Chapter 2. The system was shown in Figure 2.1 and the diagrams of maximal benefit were shown in Figure 2.3. For this case study, we did multiple simulations where we varied the order of actors in the decision process and the spin values with which the actors started the simulation. In all cases the result was the same: The simulation stabilized immediately towards the system configuration in which both Russia and Germany have their maximal benefit. Figure 5.8 shows the number of steps that resulted in a certain topological diagram. This example shows the power of the actors with high propensity values. However, it is interesting to note that, while Russia and France have the same maximal benefit, it is Russia that is able to achieve its maximal benefit because,

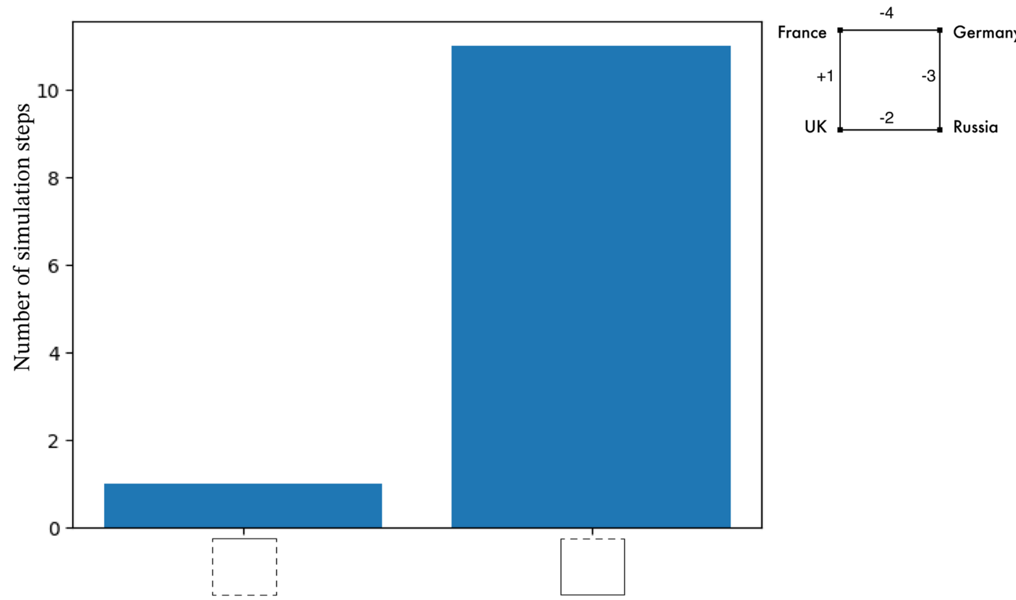


Figure 5.6: The topological diagrams of the system of Case study 2A that were created during a simulation. The vertical axis shows how many steps resulted in the creation of the topological diagrams shown on the horizontal axis. The diagram on the left is the start configuration and is never visited again during the simulation. The simulation is ended after *11* steps because the system found itself in the same configuration (the diagram on the right) for *10* simulation steps. Only Germany and Russia achieve their maximal benefit.

in contrast to France, this value is stratified over multiple connections to other actors (the UK decides to satisfy its propensity with Russia and not France because this is more beneficial). In Case study 1A, every actor was able to achieve its maximal benefit. This time, this is not possible due to differences in size of the propensities. The UK can only achieve a local maximal benefit by satisfying its propensity with Russia, denying France a maximal benefit in the process.

5.3.2 Case study 2B: All actors possess extended rationality

For the case with extended actors only, we did multiple simulations where we varied the order of the actors in the decision making process and the spin values with which the actors started the simulation. The result is equal to that of the previous section: the system stabilizes towards a system configuration in which Germany and Russia achieve a maximal benefit. This is, as in the previous case, linked to the connection of France and Russia with the UK. The UK, being the last actor, cannot find a path towards a maximal benefit configuration and favours satisfying its propensity with Russia over its propensity with France (due to the size difference of these propensities). It is therefore Russia that achieves its maximal benefit instead of France. Once again, the lack of power of the actor with the lowest maximal benefit is shown.

5.3.3 Case study 2C: Only one actor possesses extended rationality

For the case with only one extended actor, we varied the order of the actors and the spin values with which they started the simulations. We also varied the actor possessing extended rationality. The result was still equal to that of the all-limited case. This goes to show the power the actor with the highest maximal benefit has but it is also interesting to note the role of the actor with the lowest maximal benefit. This actor is not able to achieve a maximal benefit but it is able to grant another actor to which it is connected its maximal benefit. This makes this actor an interesting subject of diplomacy for these actors to which it is connected.

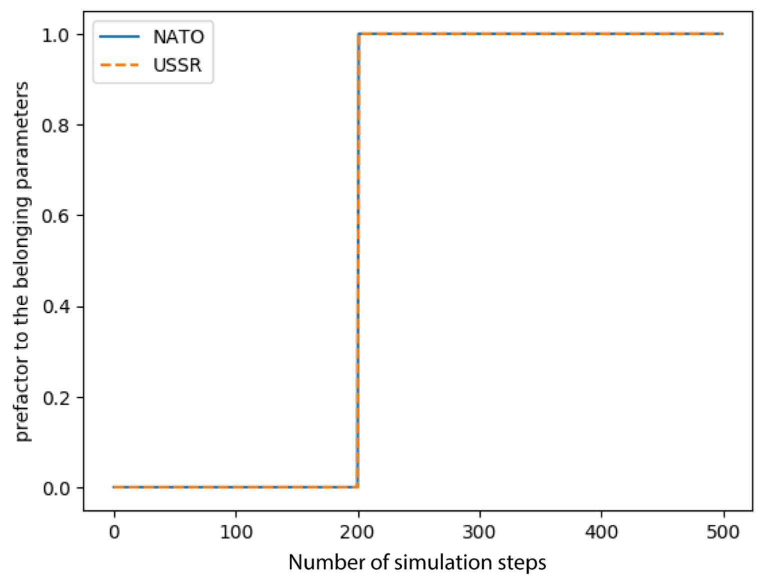


Figure 5.7: By multiplying the belonging parameters with the prefactor functions shown, the global principle in the Balkan system of Case study 3A is suppressed until simulation step *200* and the actors interact on historical propensities. Before the first substep of round *200*, the global principle emerges as the belonging parameters take their original value. Greece and Turkey adopt a belonging parameter $\varepsilon = +1$, putting them in the NATO (blue). Bulgaria, Serbia and Romania adopt a belonging parameter $\varepsilon = -1$ putting them in the USSR (orange).

5.4 Case study 3: Global principle alliances

5.4.1 Case study 3A: The emergence of a global principle

To study the effect of global principles on the decision making process of actors, we use the Balkan example from Chapter 3. All actors possess extended rationality. This system is, without global principles at play, inherently unstable because of the extended rationality of the actors. The system is shown in Figure 3.1 and the topological diagrams for the system without global principle were shown in Figure 3.4. The propensities and belonging parameters introduced by the global principle are the same ones as in Chapter 3 (The propensities due to the global principle have size $J = 10$, Greece and Turkey have a belonging parameter $\varepsilon = +1$. Bulgaria, Serbia and Romania have a belonging parameter $\varepsilon = -1$). The global principle emerges after *200* simulation steps. This is done by multiplying the belonging parameters of all actors with the prefactor functions shown in Figure 5.7: Before step *200*, the global principle is suppressed because the belonging parameters are all zero due to these prefactor functions. Before the first substep of step *200*, the belonging parameters are restored to their original value and the global principle emerges. Figure 5.8 shows how many steps resulted in the creation of a certain topological diagram before the rise of the global principle (One can consult Figure 3.4 for reference). It is clear that the diagrams, or coalition distributions, in which two actors take on the other three are preferred by the system. Figure 5.9 shows the evolution of the system. It can be seen that the global principle interactions rule the system immediately after emerging and the system instantly settles into the stable configuration in which all countries align their spin value with their belonging parameter.

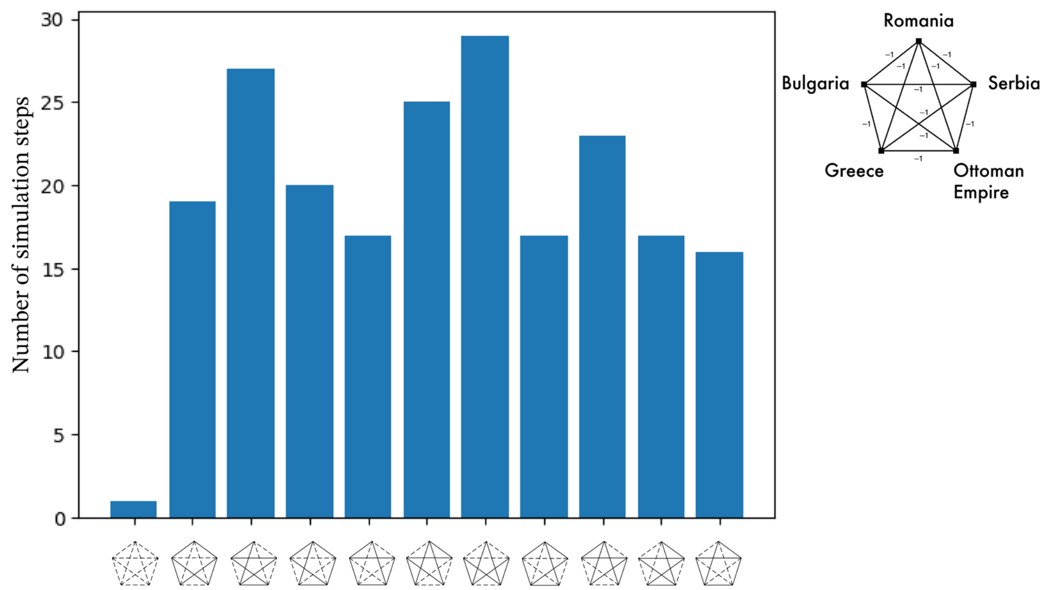


Figure 5.8: The topological diagrams of the system of Case study 3A that were created during a simulation before a global principle emerged after *200* steps. The vertical axis represents the number of steps that resulted in the creation of the topological diagrams shown on the horizontal axis. The diagram on the left represents the starting configuration in which all actors have the same spin value

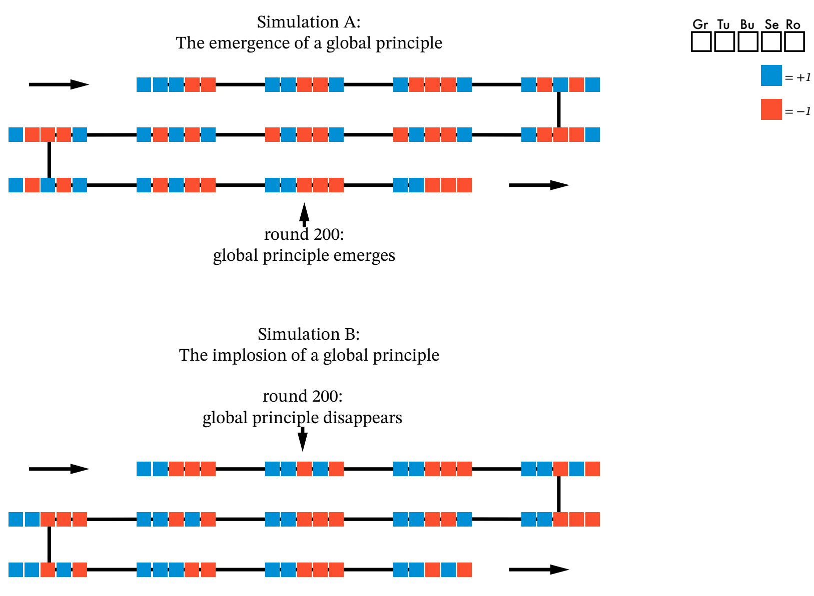


Figure 5.9: The time evolution of the Balkan system of Case study 3A and 3B when the global principle emerges (A) and when the global principle disappears for the last three actors (B). A blue rectangle represents a spin value of $\sigma = +1$ and a red rectangle represents a spin value of $\sigma = -1$

5.4.2 Case study 3B: The dissolution of a global alliance

To simulate an implosion of a global alliance, we keep the system setup from Case study 3A but use the prefactor functions shown in Figure 5.10. One prefactor function alters the belonging parameters of Bulgaria, Serbia and Romania in such a way that the global principle, for these countries, disappears before the first substep of the *200th* step. After this, they interact with each other and with Greece and Turkey based on their historical propensities. The other prefactor function preserves the original values of Greece and Turkey's belonging parameters. They keep interacting with each other based on their global principle propensity. Figure 5.11 shows how many steps resulted in the creation of a certain topological diagram after the dissolution of the Soviet Union (USSR). One can see that, even with an extended rationality, Greece and Turkey remain in a stable alliance while the other actors start switching alliances one by one in order to maximize their benefit. To illustrate this point further: Figure 5.9 shows the evolution of the system from a stable system into a partly unstable system.

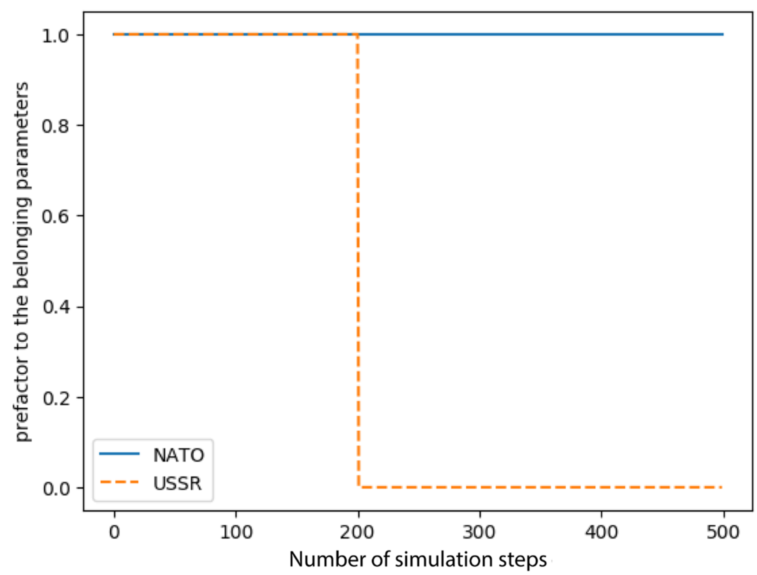


Figure 5.10: By multiplying the belonging parameters with the prefactor functions shown, the global principle in the Balkan system of Case study 3B is suppressed after simulation step *199* for Bulgaria, Serbia and Romania. Before the first substep of round *200*, the global principle disappears for these three actors as the belonging parameters take the value $\varepsilon = 0$ and they interact with the other actors based on historical propensities. Greece and Turkey keep their belonging parameter $\varepsilon = +1$ and keep interacting with each other based on the global principle propensity.

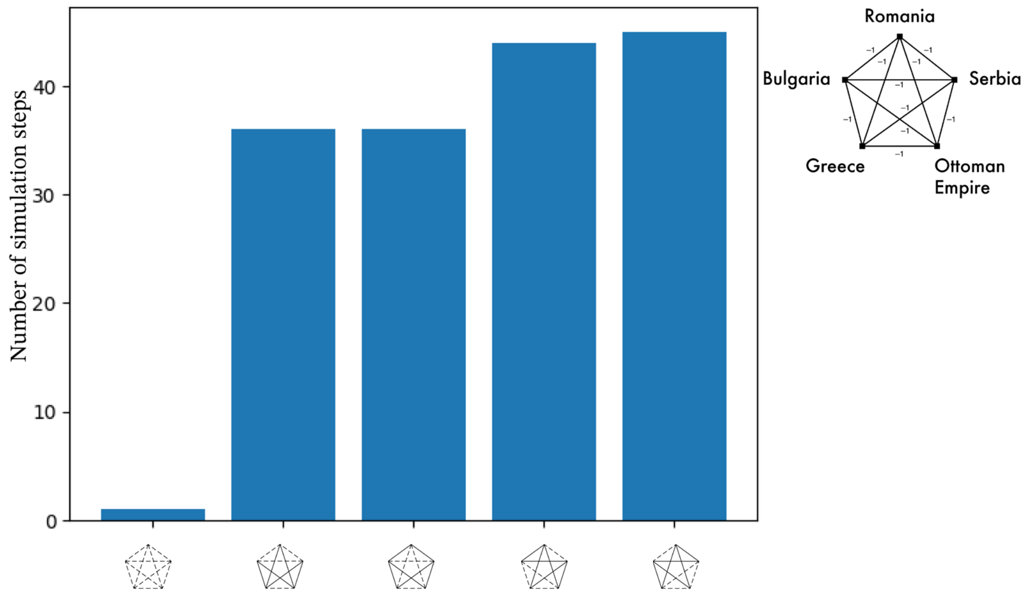


Figure 5.11: The topological diagrams of the Balkan system of Case study 3B that were created in the simulation steps after the global principle disappeared for the actors of one of the two coalitions. The belonging parameters of Bulgaria, Serbia and Romania went to zero. The belonging parameters of the actors (Greece and Turkey) of the other global alliance remained were not affected. Greece and Turkey keep their alliance, the other tree actors continuously switch coalitions in order to improve their benefit.

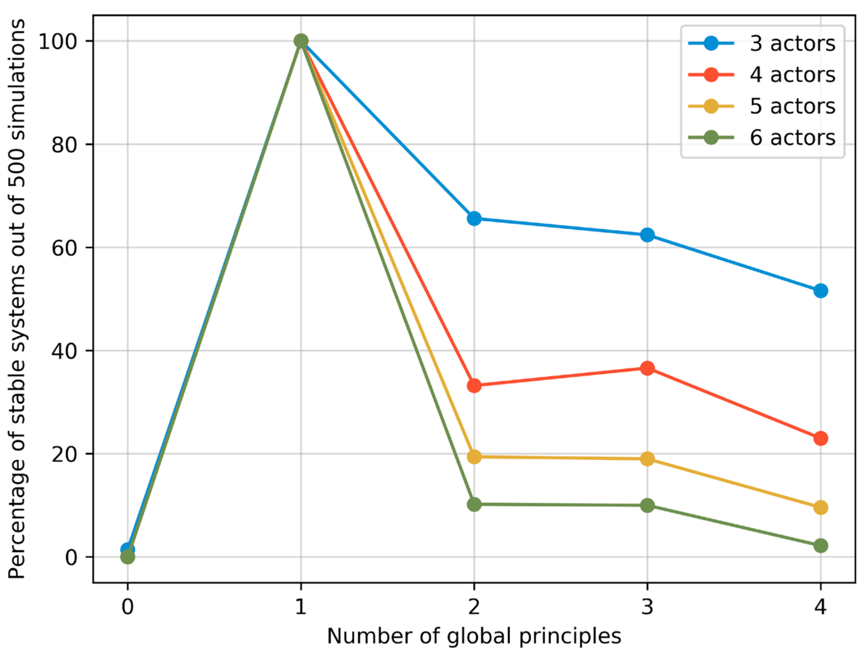


Figure 5.12: The percentage of stable systems in function of the number of global principles and actors. For each combination of number of actors and number of principles, *500* different systems are generated in which the belonging parameters of the actors on the global principles are chosen randomly. These systems are then simulated for *50* simulation steps. If the systems are not stabilized after *50* steps, they are considered unstable.

5.5 Case study 4: The stability of multi-global principle systems

To study the stability of systems with multiple global principles we created a tool that could generate a fully connected (i.e. all actors are connected) system with a specified number of global principles and actors. The belonging parameters of the actors to the various global principles are chosen randomly with a random number generator. The propensities of the global principles were chosen to be equal and larger in size than the historical propensities. The historical propensities were chosen to be all negative and equal in size as well. With the simulation software developed in the context of this work, one can study the effect of the number of actors and the number of global principles on the ability of the system to stabilize. *20* combinations of number of actors and number of global principles were created. The number of actors ranges from three up to six actors and the number of global principles ranges from zero up to four. For each combination of number of actors and number of global principles, *500* systems were generated. In those systems, the belonging parameters on the principles of the actors were randomly chosen. The systems were simulated for *50* steps. If the same system configuration is created in *10* consecutive steps, the system is assumed to be stable. In the simulations where no stable configuration was emerging after *50* simulation steps, the system was considered to be unstable. The results are shown in Figure 5.12. Because of how we chose the historical propensities, it is not surprising that the systems with no global principles were all unstable. It is also not surprising that the systems with only one global principle were all stable. They are stable because we chose the propensities of the global principle in such a way that the global principle would be more important than the historical propensities. It was shown in Chapter 1 and Chapter 3 that such systems were always stable. Adding global principles, we see that the systems are more difficult to stabilize and the drop in stable systems between the first and second global principle becomes steeper when there are more actors in the system. This could be attributed to the simulation time: Because of computational limitations, the simulation time for every simulation had to be limited to *50* steps of decision making. For systems with a large number of actors and global principles, *50* simulation steps may not be enough for the system to find a stable configuration. One reason could be that there are various stable system configurations. When each actor attempts to bring the system in a different stable system configuration, it complicates the stabilization process. However, there is still something to learn from the drop off between the second and fourth global principle. It is significantly smaller than the decrease between the first and the second principle. This effect could be attributed to the fact that a multi-global principle system can always be simplified to a system with only two global principles as was shown by Vinogradova and Galam in [9]: For actors i and j, one can introduce a new global principle α for which both actors have a belonging parameter $\varepsilon_\alpha = +1$. The size of the propensity due to this global principle is the sum of the original propensities for which the actors both had a positive belonging parameter. Another new global principle β, associated with a belonging parameter of $\varepsilon_\beta = -1$ for both actors, unifies the propensities for which the actors had negative belonging parameters.

Chapter 6 Future work: The United States presidential elections

6.1 Introduction

This chapter knits together the different concepts developed in chapters 1-5 and hopes to expand the list of use-cases of the Natural model of coalition forming with a new application: Making forecasts about the presidential elections of the United States. This concept also aims to show the full potential of the Natural model and all of its extensions. For the development of this application of the Natural model, we were inspired by many existing election models, in particular the work from Castellano et al (2009) and the website FiveThirtyEight, known for its election forecasts, proved very useful [22, 23]. When mapping the United States presidential elections onto a Natural model system, one needs to identify the extended actors. The spin value of an actor represents the vote of this actor for one of the two political parties while a third spin value is added for actors who do not vote. Key issues in the election and their effect on the voting blocs could be modeled by the global principles of the Global alliance model extension. Finally, the local magnetic fields of the diplomacy extension could be used to model the effect of the media, the incumbency of the president and the economy.

6.2 Mapping the United States election on the Natural model of coalition forming

With the presidential elections being indirect elections, the model should consist of fifty subsystems, each modeling the election in a different state. For every such subsystem, we need to identify the key extended actors and their historical propensities, the global principles and the propensities these global principles generate and the local magnetic fields at play. By developing subsystems that model the different states, it is possible to take into account the cultural and demographic differences between the populations from different states. Fore example, the whites without college degree make up half of the voting population in swing states such as Wisconsin and Pennsylvania in 2016. They played a major role in turning these states Republican and it is crucial that this group is represented as an actor in the Natural model system of these states [24]. In California, on the other hand, the Hispanics are the largest single ethnic group and since they overwhelmingly vote Democratic, one definitely

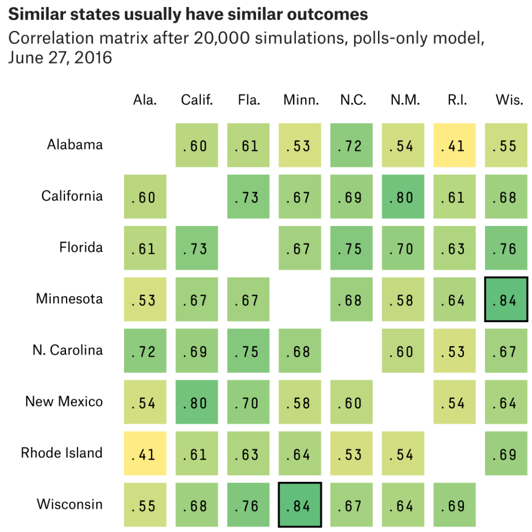


Figure 6.1: The demographic correlations between the election outcomes of different states as calculated by the election model of FiveThirtyEight. States with a similar population, such as Wisconsin and Minnesota, also have similar election results simulated by the model. Datavisualization made by FiveThirtyEight [23]

needs to include them in California's Natural model system in order to make a proper forecast of the election in this state [25, 26]. Both these examples should make it clear that the choice of which actors to put in a subsystem, is dependent on the demographics of the state modeled by the subsystem. For example: Blacks, Hispanics, LGBTQ, evangelicals, seniors and the white working class are possible actors in a subsystem. Note that the actors in a subsystem can only interact with other actors from its subsystem. Actors in a subsystem interact with each other based on their historical propensities from the Natural model (see Chapter 2) and the propensities due to global principles such as health care, economic policy, immigration and gun reform from the Global alliance model (see Chapter 3).

There are some effects that influence the country as a whole: a good economy generally favours the incumbent party and the country tends to give a sitting president another term (the last one-term president was George H. W. Bush) [27]. Another country-wide effect is the effect the media has on voters. In the current climate of polarization, the media tends to strengthen the opinions voters already have. All these effects can be taken into account by introducing (local) magnetic fields, of the magnetic field-extension discussed in Chapter 3, with which actors interact. Finally, the election results from states with similar demographics are usually correlated. Figure 6.1 (FiveThirtyEight, 2016) shows the correlation between the election outcomes of the different states as calculated by FiveThirtyEight's election model [23]. To incorporate this correlation effect, we introduce a magnetic field that is generated by the magnetization (the sum of the spin values) of other states weighted with the similarity between the states demographics. The benefit for a voting group i in a state A then becomes

$$B_{i,\ State\ A} = \sum_{j \neq i}^{N} [J_{ij} + \sum_{\alpha}^{k} G_{\alpha,ij}\epsilon_{\alpha,i}\epsilon_{\alpha,j}]\sigma_i\sigma_j + \sigma_i|\beta_{Fox}| + \sum_{B}^{50} b_B\sigma_i, \tag{6.1}$$

where

1. J_{ij} is the historical propensity between actors i and j.
2. $G_{\alpha,\,ij}$ is the propensity between actors i and j because of key issue α
3. $\varepsilon_{i,\,\alpha}$ is the belonging parameter of actor i on key issue α
4. $\sigma_i|b_{Fox}|$ is the field generated by the effect of the media on voters.
5. $\sum_B^{50} b_B$ is the magnetic field generated by other states B.

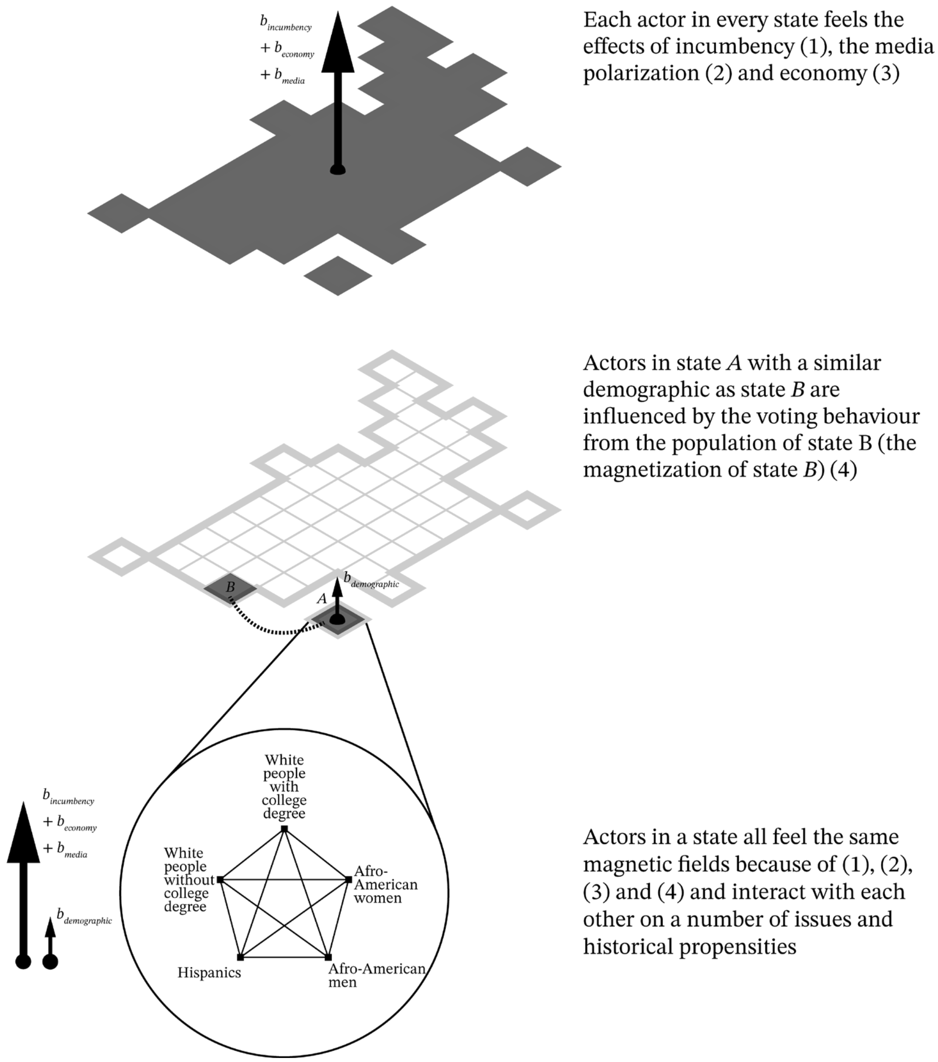


Figure 6.2: Concept of the adaptation of the Natural model of coalition forming for the presidential elections of the United States. The subsystem that models the elections in Hawaii consists of five actors which interact with each other on global principles and historical propensities. (The five actors in the subsystem are to illustrate the model and are not chosen because of research.) The five actors in the subsystem are influenced by the voters in California because both states have a similar demographic. They also feel the effect of the incumbency, the economy and the media.

When modeling elections based on the behaviour of demographic groups, one should refrain from treating these groups as monolithic blocks: Not all 'whites without a college degree' will vote for the Republican party in states such as Wisconsin. This dynamic is perhaps already present in the model. Due to the high amount of parameters (the multiple global principles (see discussion in Section 5.5), the multiple magnetic fields etc.) and the extended rationality of the actors, we can assume that the subsystems are unstable. They will fluctuate between the different topological diagrams associated with maximal benefits for the actors. This would resemble Case study 1B with all extended actors (see section 5.2.2). Looking at Figure 5.4, we can use the amount of simulation steps a subsystem 'spends' in each topological diagram as a way to approximate the occurrence probability of those topological diagrams. This occurrence probability of a topological diagram allows us to calculate how many simulation steps an actor ends up in the Democratic coalition or the Republican coalition. We can use this to approximate the voting behaviour of the voters of the demographic group associated with this actor (I.e. in State *A*, percentage *X* of the actor 'whites without college degree' will vote Democratic, percentage *Y* will vote Republican). The winning coalition in a state is then determined by taking into account the demographic size of the actors along with their voting behaviour. The occurrence probabilities of the topological diagrams give us statistics rather than the winner: It allows us the declare with a certain confidence interval who will win the election in each state. Since only about *50* percent of the eligible people vote in the election, the disenfranchised voter who does not feel represented by either political party and does not participate in the election is an important dynamic the model should possess [28]. To incorporate the disenfranchised voter into the model, a third spin value should be added to the model such that actors can now choose between the values [$-1, 0, +1$] to maximize their gain. Choosing the spin value $\sigma = 0$, could be interesting for an actor i if both the other choices lead to negative benefits. This way, actor i does not participate in the process and only has a benefit of $B_i = 0$ but it in choosing this spin value it also avoids an even lower, negative benefit. One should note that the spin value $\sigma = 0$ can erase the possibility of frustration for the associated actor.

One important caveat of this framework is that, although it allows for a thorough modeling of the elections on a state-by-state basis, it also introduces a lot of parameters with corresponding uncertainty. A lot of data is needed to fix these parameters. Getting this data requires a new approach to data gathering in the context of elections. Rather than asking voters which party they prefer, one should quiz them on their stance regarding the issues that define the election and their interactions with other demographics on these issues. Census data is also necessary to determine the size of each demographic voting group in all the states. In light of this discussion, it should be noted that, currently there does not yet exist a method of calculating propensities based on real life data. This remains to be done in future research.

Conclusion

The first part of this dissertation provided a synopsis of the currently existing theoretical work on the use of spin glasses to study the formation of coalitions among actors. The dynamics of two models of coalition formation were studied in particular: The Natural model of coalition forming and the Global alliance model, both established by Vinogradova and Galam in a series of papers [8, 9, 10]. It was shown that the Global alliance model always yielded a stable system when the propensities due to the global principle were larger in size than the historical propensities of the Natural model. It is in this scope that we showed that the dynamics and instabilities after the Cold War in the Balkan region could captured by a Natural model system extended by a single global propensity. Some additional extensions to these models, built upon the physical concepts of magnetic fields and temperature, were also discussed. The second part of this work consisted of developing software to simulate Natural model systems. We simulated the systems put forward in the first part as examples and showed that the dynamics of the Natural model and the Global alliance model are indeed present in the simulation software. The stability of the Global alliance model under various circumstances was studied as well. We have shown that the Global alliance model becomes increasingly unstable when more global principles and actors are introduced. One of the reasons for this behaviour could be that the simulation needs more simulation steps in order to stabilize into a stable configuration. Due to computational constraints, this hypothesis could not be tested. Finally, we proposed a way the framework of the Natural model and its computational implementation could be used to make predictions with regard to the outcome of upcoming presidential elections in the United States. However, at this stage such an ambitious agenda is out the scope of the present thesis. A good deal of issues still needs to be resolved, in particular, the need for a proper way to evaluated propensities between actors based on the available data provided by real life social interactions. We hope it could become the scope of future research.

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