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Effect of a Zeeman field on the transition from superconductivity to ferromagnetism in metallic grains

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Abstract – We investigate the competition between pairing correlations and ferromagnetism in small metallic grains in the presence of a Zeeman field. Our analysis is based on the universal Hamiltonian, valid in the limit of large Thouless conductance. We show that the coexistence regime of superconducting and ferromagnetic correlations can be made experimentally accessible by tuning an external Zeeman field. We compare the exact solution of the model with a mean-field theory and find that the latter cannot describe pairing correlations in the intermediate regime. We also study the occurrence of spin jumps across the phase boundary separating the superconducting and coexistence regimes.

Introduction. – The hallmark of the BCS model of superconductivity in metals is the presence of an excitation gap Δ. This gap is caused by the formation of Cooper pairs describing correlated electron pairs in time-reversed states. Thus, pairing correlations in superconductors tend to minimize the total spin of the electron system. Ferromagnetic correlations, on the other hand, prefer to maximize the total spin and form a macroscopic magnetic moment. Early work [1–5] predicted a state in which both pairing and ferromagnetic order are present, if ferromagnetism is caused by localized paramagnetic impurities. The experimental observation that both states of matter can coexist in heavy-fermions systems [6–8] and high-T_c superconductors [9,10] came as a surprise and led to the search for new theoretical models to describe this coexistence. A BCS-like model of s-wave pairing combined with a simple Stoner-like model of ferromagnetism was used to derive such an intermediate state within a mean-field approximation [11]. However, it was argued that such a state is unstable in the bulk [12–14]. Furthermore, it was shown that a proper Hartree-Fock mean-field theory of the model does not support coexistence of s-wave superconductivity and ferromagnetism [15,16].

A similar model of BCS-like pairing and exchange interaction, known as the universal Hamiltonian, was shown to be valid in small metallic grains in the mesoscopic regime for a Thouless energy E_T that is large compared with the single-particle mean-level spacing δ [17,18]. In such a finite-size system, a partially paired state with finite spin polarization exists within a narrow parameter regime [19]. Since this coexistence regime is relatively small, it would be difficult to observe it experimentally. It has been suggested that the probability of spin polarization in the presence of pairing correlation may be enhanced by mesoscopic fluctuations [20] or by an asymmetric spin-dependent bandwidth of the single-particle spectrum [19].

Here we study the competition between ferromagnetic and pairing correlations in metallic grains in the crossover regime from a few-electron system ($\Delta \ll \delta$) to the bulk ($\Delta \gg \delta$). While a strong-coupling phase whose fluctuation properties are different from the universal Hamiltonian was derived in ref. [21], here we assume the weak coupling limit in which the universal Hamiltonian description is valid. We use Richardson's solution of the BCS-like interaction [22] and the known solution of the exchange model [23] to determine the ground state of the grain. For sufficiently small grains, there is a regime in pairing gap Δ/δ and exchange coupling J_s/δ, in which the ground state is partly paired and partly polarized. We show that, in the presence of a Zeeman field, the exchange coupling at which the crossover from a pure superconducting state to the coexistence regime takes place decreases to values that can be realized in several metals. The onset of magnetization with increasing exchange coupling at

a given pairing gap corresponds to a spin jump $\Delta S \geqslant 1$, followed by successive spin increments of $\Delta S = 1$. The magnitude of the initial spin jump depends on the value of Δ/δ. Similar spin jumps were found in the crossover from a superconducting state to a paramagnetic state [24,25], where they are reminiscent of a first-order transition in the bulk. We apply a mean-field theory similar to the one used in ref. [25] and compare with the exact results. In contrast to the exact solution, we find that the mean-field approximation cannot describe pairing correlations in the intermediate regime of partial spin polarization.

Model. – An isolated metallic grain in which the single-particle dynamics are chaotic (or diffusive) and whose dimensionless Thouless conductance $g_T = E_T/\delta$ is large ($g_T \gg 1$), can be described by an effective universal Hamiltonian [17,18]

$$\hat{H} = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - G\hat{P}^\dagger \hat{P} - J_s \hat{\mathbf{S}}^2 + g\mu_B H \hat{S}_z \,. \quad (1)$$

Here $c^\dagger_{k\sigma}$ is the creation operator for an electron in the single-particle level ϵ_k with either spin up ($\sigma = +$) or spin down ($\sigma = -$). The one-body term in (1) describes the kinetic energy plus confining single-particle potential. The second term on the r.h.s. of eq. (1) is a pairing interaction with strength G and where $P^\dagger = \sum_i c^\dagger_{i+} c^\dagger_{i-}$ is the pair creation operator. The third term in (1) is an exchange interaction expressed in terms of the total spin operator $\hat{\mathbf{S}} = \frac{1}{2}\sum_{k\sigma\sigma'} c^\dagger_{k\sigma} \boldsymbol{\tau}_{\sigma\sigma'} c_{k\sigma'}$ (τ_i are Pauli matrices). The parameter J_s is the exchange coupling constant (estimated values of J_s for a variety of materials were tabulated in ref. [26]). The inclusion of such an exchange interaction in quantum dots [27] explained quantitatively the measured peak height and peak spacing statistics [23,28]. The last term on the r.h.s. of eq. (1) describes the coupling of an external Zeeman field H (applied in the z-direction) to the spin of the dot. Here g is the g-factor of the electrons in the grain (taken to be positive) and μ_B is the Bohr magneton. Orbital diamagnetism can be neglected for small grains [25]. The charging energy $e^2\hat{N}^2/2C$ (C is the capacitance of the grain) is a constant for a grain with a fixed number of electrons N and was omitted in the Hamiltonian (1).

In this work, we do not consider mesoscopic effects that originate in the random matrix description of the single-particle Hamiltonian. To construct a typical phase diagram of a single grain, we consider a generic equidistant spectrum $\epsilon_k = k\delta$ with $-N_o \leqslant k \leqslant N_o$ at half-filling. Thus we have $N = 2N_o$ for an even number of electrons ($p = 0$) and $N = 2N_o + 1$ for an odd number ($p = 1$).

Exact solution. – In the absence of a pairing interaction ($G = 0$), the Hamiltonian (1) can be solved in closed form [23]. The orbital occupations $\hat{n}_k = \hat{n}_{k+} + \hat{n}_{k-}$ commute with $\hat{\mathbf{S}}^2$ and are good quantum numbers. The empty ($n_k = 0$) and doubly occupied ($n_k = 2$) orbitals do not contribute to the total spin, so the total spin of the grain is obtained by coupling the singly occupied levels with spin 1/2 to total spin S and spin projection M. For a specific set $\mathcal{B}$ of b singly occupied levels, the total spin ranges from $S = p/2$ to $S = b/2$ with each spin value having a degeneracy of $d_b(S) = \binom{b}{S+b/2} - \binom{b}{S+1+b/2}$. A complete set of eigenstates is then given by $|\{n_k\}, \gamma, S, M\rangle$ where γ denotes a set of quantum numbers distinguishing between eigenstates with the same spin and orbital occupation numbers [23,29].

The pairing interaction can only scatter time-reversed pairs from doubly occupied to empty orbitals but does not affect the singly occupied levels (referred to as "blocked" levels). It is therefore sufficient to diagonalize the reduced BCS Hamiltonian $\sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - GP^\dagger P$ using the single-particle subspace $\mathcal{U}$ of empty and doubly occupied levels. This problem was solved by Richardson [22]. The eigenenergies are given by

$$E_m = \sum_{\mu=1}^{m} E_\mu \,, \quad (2)$$

where E_μ are parameters that characterize the eigenstate and $m = (N-b)/2$ is the number of pairs. Richardson's parameters E_μ are found by solving the set of m coupled non-linear equations

$$\frac{1}{G} + 2\sum_{\substack{\nu=1 \\ \nu\neq\mu}}^{m} \frac{1}{E_\nu - E_\mu} = \sum_{i\in\mathcal{U}} \frac{1}{2\epsilon_i - E_\mu} \quad (\mu = 1, \ldots, m) \,. \quad (3)$$

For each set of m doubly occupied levels at $G = 0$, there is a unique solution for Richardson's parameters at $G \neq 0$. We solve eq. (3) using the method of ref. [30].

The eigenstates constructed from the subset $\mathcal{U}$ of empty and doubly occupied levels have spin zero, so the total spin S of the grain is determined by the spin-(1/2) coupling of the singly occupied levels in $\mathcal{B}$. The eigenstates of the full Hamiltonian (1) are then given by $|\mathcal{B}, \{E_\mu\}, \gamma, S, M\rangle$ with energies of

$$E = E_m + \sum_{k\in\mathcal{B}} \epsilon_k - J_s S(S+1) + g\mu_B H M \,. \quad (4)$$

In this work, we focus on the ground state of the grain as a function of the interaction couplings G and J_s. To that end, we find the lowest energy $E(S)$ in (4) for a given spin S and then minimize $E(S)$ with respect to S.

The energy $E(S)$ is determined by choosing the set $\mathcal{B}$ of $b = 2S$ singly occupied levels to be closest to the Fermi energy and then solving for the lowest Richardson's state in the remaining $N - b$ levels. To show that this choice of $\mathcal{B}$ provides the lowest-energy solution for a given spin, we calculated the energy cost $\Delta E(S)$ to promote an electron in the highest singly occupied level one level up. This energy cost is shown in fig. 1 as a function of Δ/δ for $S = 1/2, 1$ and $3/2$. While pairing correlations reduce $\Delta E(S)$, the latter remains positive for all values of Δ/δ.

The physical parameter describing the pairing Hamiltonian is Δ/δ, where Δ is the bulk pairing gap and δ the single-particle mean-level spacing. The low-energy spectrum of the grain (for $J_s = H = 0$) is determined by the

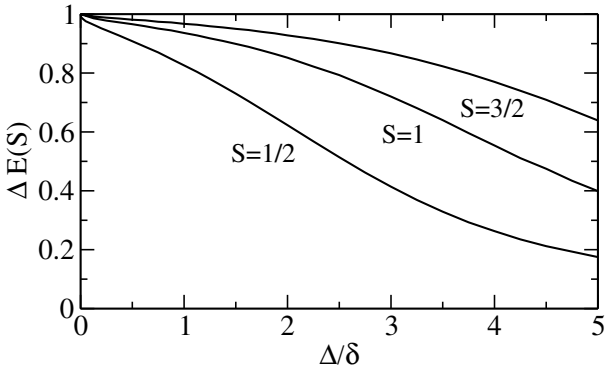

Fig. 1: The energy cost $\Delta E(S)$ (in units of δ) to promote an electron one level up from the highest singly occupied level, when the starting state is the lowest state of (1) with $b = 2S$ singly occupied levels chosen closest to the Fermi energy.

value of this parameter. We can truncate the total number of levels from N_o to $N_r < N_o$, and renormalize G such that the low-energy spectrum of the grain remains approximately the same. For a picket fence spectrum, the renormalized coupling constant is given by

$$\frac{G_r}{\delta} = \frac{1}{\operatorname{arcsinh}\left(\frac{N_r+1/2}{\Delta/\delta}\right)}\,. \tag{5}$$

Strictly speaking, this holds in the absence of an exchange interaction. However, since the exchange interaction affects only the blocked levels, we expect the renormalization (5) to hold as long as the number of blocked levels is small compared with the total number of levels in the band. The quality of this renormalization procedure depends on the choice for N_r and was discussed in detail in ref. [31].

Mean-field approximation. – We compare the findings based on the exact solution with a mean-field theory. The mean-field approach is based on a trial wave function of the form [25]

$$|\psi_S\rangle = \prod_{k\in\mathcal{B}} c^\dagger_{k-} \prod_{j\in\mathcal{U}} \left(u_j^{(S)} + v_j^{(S)} c^\dagger_{j+} c^\dagger_{j-}\right)|0\rangle \tag{6}$$

with the normalization condition $(u_j^{(S)})^2 + (v_j^{(S)})^2 = 1$. The wave function ψ_S has $b = 2S$ singly occupied levels with spin-down electrons (set $\mathcal{B}$) chosen to be closest to the Fermi energy, and is of the BCS form within the remaining set of levels $\mathcal{U}$. The lowest state with spin S is found by minimizing the expectation value $\langle\psi_S|(\hat{H} - \mu\hat{N})|\psi_S\rangle$ with respect to the variational parameters $v_j^{(S)}$. Here μ is a chemical potential ensuring that the average number of particles is N.

The mean-field energy at fixed spin S is given by

$$E_{\rm mf}(S) = 2\sum_{k\in\mathcal{U}} \epsilon_k \left(v_k^{(S)}\right)^2 - \frac{\Delta_S^2}{G} + \sum_{k\in\mathcal{B}} \epsilon_k - J_s S(S+1) - g\mu_B H S\,, \tag{7}$$

where

$$\left(v_k^{(S)}\right)^2 = \frac{1}{2}\left(1 - \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k-\mu)^2 + \Delta_S^2}}\right), \tag{8}$$

and Δ_S is a spin-dependent pairing gap. The gap parameter and chemical potential are determined by solving the gap equation together with the particle number equation

$$\frac{2}{G} = \sum_{k\in\mathcal{U}} \frac{1}{\sqrt{(\epsilon_k-\mu)^2 + \Delta_S^2}}\,, \tag{9a}$$

$$N = 2\sum_{k\in\mathcal{U}} \left(v_k^{(S)}\right)^2 + b\,. \tag{9b}$$

For an equidistant spectrum, the chemical potential can be determined by symmetry considerations and is given by $\mu = -(1-p)\delta/2$ for $N_o\delta \gg \Delta_S$.

Here, we used the same approximations as in ref. [25] and neglected a term in the energy $E_{mf}(S)$ which is proportional to $(v_j^{(S)})^4$. The result (7) is in agreement with the leading term of an expansion in the inverse number of electrons $1/N$ [32]. Comparing (7) and (4) with $M = -S$, we see that the exchange and Zeeman terms are treated exactly in this mean-field approximation. The ground-state spin in the mean-field approximation is found by minimizing $E_{mf}(S)$ in (7) with respect to S.

Ground-state phase diagram. – The ground-state spin of the grain is determined by the competition between various terms in the universal Hamiltonian. The one-body part (kinetic plus confining one-body potential) and pairing interaction favor minimal spin $S = p/2$, while exchange interaction and Zeeman field favor a maximally polarized state. We have studied the ground-state spin as a function of the three parameters Δ/δ, J_s/δ and $g\mu_B H/\delta$. Using the exact solution, we find three different phases: a superconducting phase where the number of pairs is maximal and $S = p/2$, a ferromagnetic phase where the system is fully polarized $S = N/2$ (all electrons are with spin down), and an intermediate regime. The intermediate regime describes a partially polarized state $S < N/2$, in which $b = 2S$ electrons reside in singly occupied levels closest to the Fermi energy and the remaining electrons are paired to give spin zero.

The phase diagram in the Δ/δ-J_s/δ plane of a grain with even number of electrons and in the absence of a Zeeman field ($H = 0$) is presented in fig. 2(a). For weak pairing, the superconducting and ferromagnetic phases are separated by an intermediate regime. The boundaries of this intermediate regime are described by two critical values $J_s^{(1)}$ and $J_s^{(2)}$ of the exchange interaction that are function of Δ/δ. The critical value $J_s^{(1)}/\delta$ is a monotonically increasing function of Δ/δ, *i.e.*, a stronger exchange interaction is required to polarize a grain with stronger pairing correlations. However, $J_s^{(2)}/\delta$ is almost insensitive to Δ/δ. The intermediate regime shrinks at larger Δ/δ and eventually disappears above $\Delta/\delta \sim 3$. For stronger

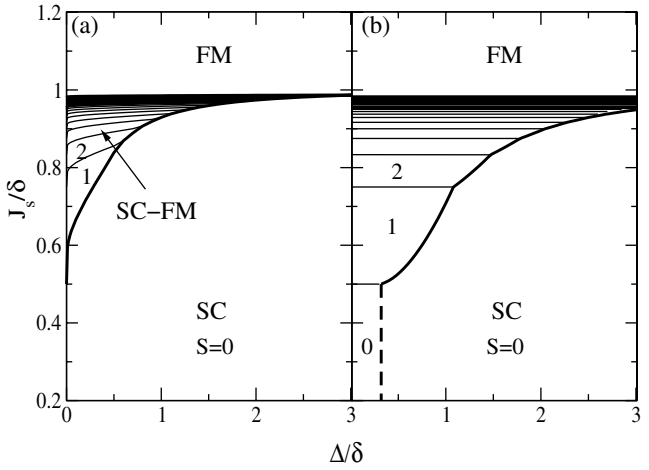

Fig. 2: Ground-state phase diagram in the J_s/δ-Δ/δ plane for an even number of electrons ($N = 50$). Left panel: exact results. Right panel: mean-field approximation (see text). The phase diagrams show a superconducting (SC) phase and a ferromagnetic (FM) phase. The exact phase diagram also exhibits an intermediate regime (SC-FM) in which the ground state is partially polarized but still has pairing correlations. The intermediate regime in the mean-field phase diagram describes a state that is a partially polarized state but does not include pairing correlations. In particular, the dashed line separates an $S = 0$ SC phase from an $S = 0$ phase with no pairing correlations. The numbers shown in the intermediate regime are the spin values in the corresponding sectors.

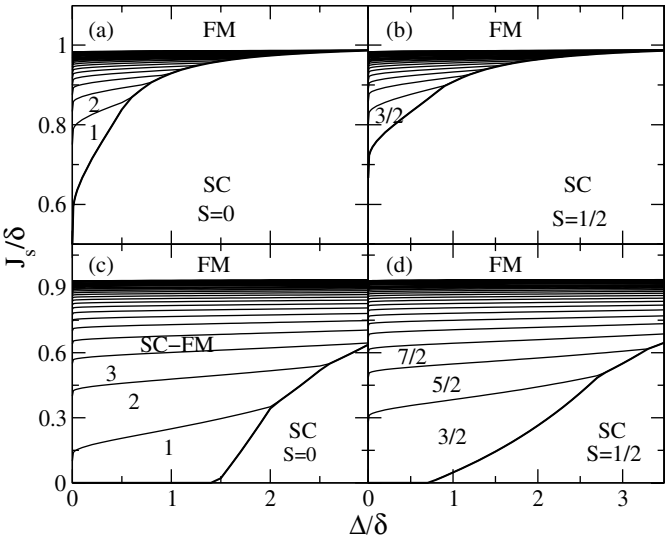

Fig. 3: Phase diagrams in the J_s/δ-Δ/δ plane at a fixed Zeeman field $g\mu_B H = 0$ (top panels) and $g\mu_B H/\delta = 2.6$ (bottom panels) for an even grain (left panels) and for an odd grain (right panels). The numbers denote the spin in each sector.

pairing correlations, the superconducting phase makes a direct transition to the ferromagnetic phase. In this regime (not shown in fig. 2), the phase boundary exhibits a strong dependence on the bandwidth N_o.

For comparison, we show the mean-field results in fig. 2(b). We observe that the mean-field results are qualitatively different from the exact solution. The region to the right of the thick solid line and dashed line describes a superconducting phase with $\Delta_0 \neq 0$. However, there is no superconducting solution (*i.e.*, $\Delta_0 = 0$) for $\Delta/\delta \leqslant 0.28$. Furthermore, in each of the partially polarized regions with spin $0 < S < N/2$, the corresponding pairing gap vanishes $\Delta_S = 0$ and no pairing correlations are present in the mean-field wave function. While solutions of $\Delta_S \neq 0$ exist for large values of Δ/δ, the actual ground state at such values of Δ/δ is a higher spin state with no pairing correlations. For example, a solution with $\Delta_1 \neq 0$ exists only for $\Delta/\delta > 2.1$ [25]. However, at this strength of pairing correlations we observe a direct transition from $S = 0$ to $S = 4$ with $\Delta_S = 0$ as the energy of the lowest $S = 4$ state with no pairing correlations is lower than the paired $S = 1$ state. As a result, the boundaries which separate different spin phases are flat, *i.e.*, independent of the pairing strength. In fact, in the mean-field approximation the ground-state wave function is a Slater determinant through the whole intermediate regime of partial spin polarization. Thus no coexistence of pairing and spin polarization is observed within the mean-field approach.

In contrast, the exact solution shows that pairing correlations are present as long as the system is not fully polarized. This can be seen in the shift of the spin transition lines to higher values of the exchange interaction strength as the pairing gap Δ/δ is increased. Thus, the exact solution predicts a regime in which pairing correlations and spin polarization coexist. In the following, we only discuss results obtained from the exact solution.

More detailed phase diagrams for $H = 0$ are shown in the top row of fig. 3 for both grains with even (panel (a)) and odd (panel (b)) number of electrons. For weak pairing we observe an odd-even effect (in number of electrons). In particular, the critical value $J_s^{(1)}$ is larger for the odd grain, even though the presence of a blocked level in the odd superconducting phase weakens pairing correlations in the odd grain. This is because increasing the spin from 1/2 to 3/2 in the odd grain costs more one-body energy than increasing the spin from 0 to 1 in the even grain.

The phase boundaries for a finite Zeeman field $g\mu_B H/\delta = 2.6$ are shown in the bottom row of fig. 3. The effect of a Zeeman field is twofold. First, it helps polarizing the grain, making the value of $J_s^{(1)}$ for a given pairing gap smaller. Second, at given exchange strength J_s/δ, it increases the critical value of Δ/δ at which partial spin polarization is destroyed. Both effects together increase the size of the intermediate regime in the Δ/δ-J_s/δ plane.

Spin jumps. – As we increase the exchange coupling constant J_s/δ at fixed Δ/δ and Zeeman field, the spin increases by discrete steps from its minimal value $S = p/2$ to its maximal value of $S = N/2$. In the absence of pairing ($\Delta = 0$), the transition from spin S to spin $S+1$ occurs

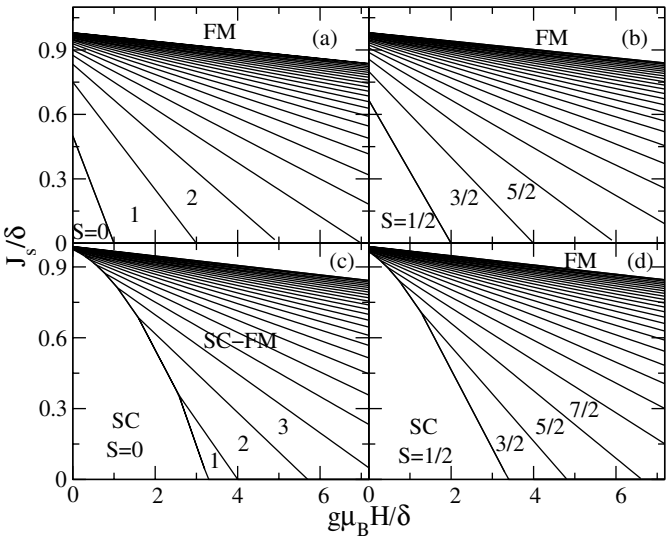

Fig. 4: Phase diagrams in the J_s/δ-$g\mu_B H/\delta$ plane at fixed $\Delta/\delta = 0$ (top row) and $\Delta/\delta = 2$ (bottom row) for an even grain (left column) and for an odd grain (right column). Numbers denote the spin in each sector.

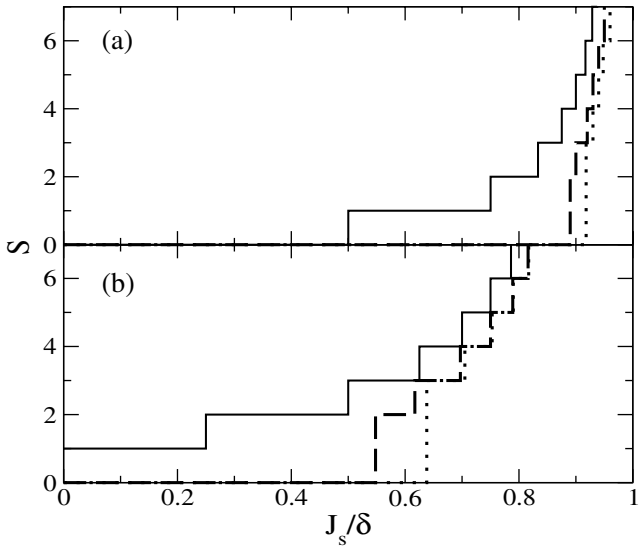

Fig. 5: Ground-state spin *vs.* exchange coupling J_s/δ for an even grain at a fixed Zeeman field strength $g\mu_B H = 0$ (top panel) and $g\mu_B H/\delta = 2$ (bottom panel). Solid lines correspond to a grain with no pairing correlations $\Delta/\delta = 0$. The dashed lines describe staircase functions with a spin jump of $\Delta S = 2$ for $\Delta/\delta = 0.7$ (top) and $\Delta/\delta = 2$ (bottom). The dotted lines correspond to staircases with a spin jump of $\Delta S = 3$ for $\Delta/\delta = 0.9$ (top) and $\Delta/\delta = 2.3$ (bottom).

for an exchange coupling of

$$J_s/\delta = \frac{(2S+1) - g\mu_B H/\delta}{2S+2} \qquad \text{at} \quad \Delta = 0\,. \tag{10}$$

The corresponding $\Delta = 0$ phase diagrams in the J_s/δ-$g\mu_B H/\delta$ plane are shown in figs. 4(a) and (b) for even and odd grains, respectively. In particular, the phase boundaries are given by $J_s^{(1)} = \delta(p+1)/(p+2) - g\mu_B H/(p+2)$ and $J_s^{(2)} = \delta(N-1)/N - g\mu_B H/N$. The ground-state spin increases as a function of J_s in steps of $\Delta S = 1$. An interesting qualitative change in the presence of pairing correlations is the possibility of a spin jump $\Delta S > 1$. For $\Delta/\delta < 0.6$, the ground-state spin still increases in steps of $\Delta S = 1$ *vs.* J_s. However, for $0.6 < \Delta/\delta < 0.8$, the ground-state spin jumps from 0 to 2 for a value of the exchange coupling in the range $0.87 < J_s/\delta < 0.9$. The size of the first-step spin jump gets larger with increasing Δ/δ. All subsequent steps are of size one (see fig. 3(a)).

Similar spin jumps were observed when superconductivity breaks down due to the presence of a large external Zeeman field [25]. The experimental findings were qualitatively explained using the mean-field theory we discussed previously (but without the inclusion of an exchange interaction). It was concluded in ref. [25] that the first-order phase transition from a superconductor to a paramagnet, observed in thin films, is "softened" in metallic grains. Here we observe that spin jumps also occur in the presence of exchange correlations. In the absence of an external Zeeman field, these spin jumps are predicted to occur at exchange coupling values of $J_s/\delta > 0.87$. Such exchange coupling values are significantly larger than the values for most metals (see fig. 9 in ref. [26]). Moreover, the exchange strength is an intrinsic material property and is difficult to tune experimentally.

The regime of spin jumps can be tuned to lower and more typical values of J_s by applying an external Zeeman field. We have already seen in figs. 3(c) and (d) that a relatively weak Zeeman field increases the size of the intermediate regime. It also means that spin jumps can be observed at smaller values of the exchange strength that are accessible to experiments. This is demonstrated in figs. 4(c) and (d) where phase diagrams in the J_s/δ-$g\mu_B H/\delta$ plane are shown for a given pairing gap of $\Delta/\delta = 2$. For example, a Zeeman field of $g\mu_B H/\delta \approx 2$ is sufficient to lower the critical exchange strength for the $0 \to 2$ spin jump to $J_s/\delta \approx 0.55$ at $\Delta/\delta = 2$ (see fig. 4(c)) as compared to $J_s/\delta = 0.89$ at $\Delta/\delta = 0.7$ without a Zeeman field (see fig. 3(a)).

The idea of a Zeeman-field tuning of the values of exchange coupling where spin jumps occur is best illustrated in fig. 5, where spin staircase functions are shown *vs.* J_s/δ. In the presence of pairing correlations and in the absence of a Zeeman field, the ground-state spin staircase is shifted to the right and compressed as Δ/δ increases (see fig. 5(a)), reflecting the fact that the intermediate region shrinks (see figs. 3(a) and (b)). For an even grain with $\Delta/\delta = 0.7$, a spin jump of $\Delta S = 2$ sets in at $J_s \approx 0.89\delta$, while for $\Delta/\delta = 0.9$, a spin jump of $\Delta S = 3$ occurs at $J_s \approx 0.92\delta$. For a finite Zeeman field of $g\mu_B H/\delta = 2$, the spin staircase functions that exhibit similar spin jumps are shifted to smaller values of the exchange strength but larger values of the pairing gap (see fig. 5(b)). Spin jumps of $\Delta S = 2$ ($\Delta S = 3$) occur at $J_s/\delta = 0.55$ ($J_s/\delta = 0.64$) and $\Delta/\delta = 2$ ($\Delta/\delta = 2.3$).

In the relevant experimental situation of a fixed exchange interaction strength, the critical value of Δ/δ at which spin jumps occur as well as the size of these jumps increase at larger values of $g\mu_B H/\delta$. The ratio Δ/δ (for the given metal) can be made larger by studying a larger grain, hence reducing the mean level spacing δ.

As an example, niobium has an exchange interaction strength of $J_s/\delta \approx 0.4$ [26]. Without an external Zeeman field, the ground-state spin will be minimal ($S = p/2$) at all values of Δ/δ (see figs. 3(a) and (b)). At a Zeeman field $g\mu_B H/\delta = 1$, the ground-state spin of niobium changes from 0 to 1 at $\Delta/\delta = 0.66$. However, at $g\mu_B H/\delta = 2.6$, a spin jump of $\Delta S = 2$ occurs from 0 to 2 at $\Delta/\delta = 2.15$ (see fig. 3(c)). For these two values of Δ/δ, we can roughly estimate the corresponding critical size of the metallic grain given the bulk gap value $\Delta = 3.05\,\mathrm{meV}$ and Fermi momentum $k_F = 11.8\,\mathrm{nm}^{-1}$ of niobium. In a Fermi gas model, the mean-level spacing is related to the volume V of the grain by $\delta = 2\pi^2\hbar^2/(m_e k_F V)$, where m_e is the electron mass. Assuming a hemispheric grain with radius r, we have the relation $r_{\mathrm{Nb}} \approx 2.7\,\mathrm{nm}(\Delta_{\mathrm{Nb}}/\delta)^{1/3}$. Thus, the Hamiltonian (1) with an equidistant spectrum predicts a $0 \to 1$ spin transition for a niobium grain of radius $r \approx 2.35\,\mathrm{nm}$ and Zeeman field of $g\mu_B H = 4.62\,\mathrm{meV}$, and a $0 \to 2$ spin jump at $r \approx 3.48\,\mathrm{nm}$ and a Zeeman field of $g\mu_B H = 3.69\,\mathrm{meV}$.

Conclusion. – We have shown that there exists a small region in the ground-state phase diagram of a small metallic grain in which pairing correlations and ferromagnetism coexist. This coexistence regime becomes larger (in the J_s/δ-Δ/δ plane) and therefore more accessible to experiments in the presence of a finite Zeeman field. In particular, we propose that for a given exchange constant (determined by the material used), spin jumps can be observed by tuning a Zeeman field. We have also shown that a quantitative study of the intermediate regime requires the use of the exact solution. Furthermore, the mean-field approximation is qualitatively different in that it does not predict any pairing correlations in the intermediate regime of partial spin polarization.

In this work, we have ignored mesoscopic fluctuations and focused on a grain with an equidistant single-particle spectrum. It would be interesting to study how mesoscopic fluctuations affect the boundaries of the intermediate phase and the size of the spin jumps.

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REFERENCES

[1] Abrikosov A. A. and Gorkov L. P., *Zh. Eksp. Teor. Fiz.*, **39** (1960) 1781 (*Sov. Phys. JETP*, **12** (1961) 1243).
[2] Clogston A. M., *Phys. Rev. Lett.*, **9** (1962) 266.
[3] Chandrasekhar B. S., *Appl. Phys. Lett.*, **1** (1962) 7.
[4] Fulde P. and Ferrell R. A., *Phys. Rev.*, **135** (1964) A550.
[5] Larkin A. I. and Ovchinnikov Yu. N., *Zh. Eksp. Teor. Fiz.*, **47** (1964) 1136 (*Sov. Phys. JETP*, **20** (1965) 762).
[6] Saxena S. S. *et al.*, *Nature (London)*, **406** (2000) 587.
[7] Pfleiderer C. *et al.*, *Nature (London)*, **412** (2001) 58.
[8] Aoki D. *et al.*, *Nature (London)*, **413** (2001) 613.
[9] Tallon J. *et al.*, *IEEE Trans. Appl. Supercond.*, **9** (1999) 1696.
[10] Bernhard C. *et al.*, *Phys. Rev. B*, **59** (1999) 14099.
[11] Karchev N. I., Blagoev K. B., Bedell K. S. and Littlewood P. B., *Phys. Rev. Lett.*, **86** (2001) 846.
[12] Zhou Y., Li J. and Gong C., *Phys. Rev. Lett.*, **91** (2003) 069701.
[13] Shen R., Zheng Z. M. and Xing D. Y., *Phys. Rev. Lett.*, **91** (2003) 069702.
[14] Shen R., Zheng Z. M., Liu S. and Xing D. Y., *Phys. Rev. B*, **67** (2003) 024514.
[15] Joglekar Y. N. and MacDonald A. H., *Phys. Rev. Lett.*, **92** (2004) 199705.
[16] Blagoev K. B., Bedell K. S. and Littlewood P. B., *Phys. Rev. Lett.*, **92** (2004) 199706.
[17] Kurland I. L., Aleiner I. L. and Altshuler B. L., *Phys. Rev. B*, **62** (2000) 14886.
[18] Aleiner I. L., Brouwer P. W. and Glazman L. I., *Phys. Rep.*, **358** (2002) 309.
[19] Ying Z., Couco M., Noce C. and Zhou H., *Phys. Rev. B*, **74** (2006) 012503.
[20] Falci G., Fazio R. and Mastellone A., *Phys. Rev. B*, **67** (2003) 132501.
[21] Murhty G. and Shankar R., *Phys. Rev. Lett.*, **90** (2003) 066801.
[22] Richardson R. W., *Phys. Rev. Lett.*, **3** (1963) 277; Richardson R. W. and Sherman N., *Nucl. Phys.*, **52** (1964) 221; Richardson R. W., *Phys. Rev.*, **159** (1967) 792.
[23] Alhassid Y. and Rupp T., *Phys. Rev. Lett.*, **91** (2003) 056801.
[24] von Delft J. and Ralph D. C., *Phys. Rep.*, **345** (2001) 61.
[25] Braun F., von Delft J., Ralph D. C. and Tinkham M., *Phys. Rev. Lett.*, **79** (1997) 921.
[26] Gorokhov D. A. and Brouwer P. W., *Phys. Rev. B*, **69** (2004) 155417.
[27] Alhassid Y., *Rev. Mod. Phys.*, **72** (2000) 895.
[28] Alhassid Y. and Malhotra S., *Phys. Rev. B*, **66** (2002) 245313.
[29] Tureci H. E. and Alhassid Y., *Phys. Rev. B*, **74** (2006) 165333.
[30] Rombouts S., Van Neck D. and Dukelsky J., *Phys. Rev. C*, **69** (2004) 061303(R).
[31] Alhassid Y., Fang L. and Schmidt S., cond-mat/0702304.
[32] Yuzbashyan E. A., Baytin A. A. and Altshuler B. L., *Phys. Rev. B*, **71** (2005) 094505.